## INDEX

## A

AA triangle similarity (AA~), 490. See also Angle-angle triangle similarity
AAS triangle congruence, 352. See also Angle-angle-side triangle congruence
Abscissa, 210
Absolute value, 7-8
Acute angle, 17
Acute triangle, 25, 105
Addition
of angles, 20-21
associative property of, 4-5
closure property of, 4
commutative property of, 4
distributive property of multiplication over, 5
inequality postulates involving, 267-268
of line segments, 12-13
Addition postulate, 119-120, 267-268
Additive identity, 5
Additive inverses, 5
Adjacent angles, 145
Adjacent interior angle, 277
Adjacent sides, 380
Adjacent vertices, 368, 380
Algebra, Boolean, 34
Alternate exterior angles, 330
Alternate interior angles, 330
Altitude(s). See also Height
of cone, 456
of cylinder, 453
of pyramid, 449
of rectangle, 389
of triangle, 175
concurrence of, 317-320
Analytic geometry, 209
Angle(s), 15
acute, 17, 105
addition of, 20-21
adjacent, 145
alternate exterior, 330
alternate interior, 330
base, 25
bisectors of, 20, 103, 137, 199
central, 536-537
classifying according to their measures, 17
classifying triangles according to, 25-26
complementary, 145
congruent, 19, 103
construction of, congruent to a given angle, 197
corresponding, 330
defined, 15
definitions involving pairs of, 145-146
dihedral, 424
exterior, of, 16, 330
polygon, 276-277, 369
triangle, 277-279
formed by tangent and chord, 567-568
formed by tangent and secant, 568-571
formed by two intersecting chords, 568
inequalities involving, for triangle, 281-284
inscribed, and their measures, 552-555
interior, of, 16, 330
polygon, 368-369
linear pair of, 148
measure of, 16
naming of, 16
obtuse, 17
plane, 424
postulates of, 135-138
proving theorems about, 144-145
right, 17
sides of, 15
straight, 16, 17
subtraction of, 20-21
sum of measures of, of triangle, 347-350
supplementary, 145-146, 148-149
theorems involving pairs of, 146-148
trisection of, 604
using congruent triangles to prove congruent, 178-179
vertex of, 15,25
vertical, 149
Angle addition postulate, 120
Angle-angle-side triangle congruence, 352. See also AAS triangle congruence
Angle-angle triangle similarity, 490. See also AA triangle similarity
Angle bisector(s)
concurrence of, of triangle, 364-365
of triangle, 176
Angle measure, preservation of
under dilation, 497
under glide reflection, 244
under line reflection, 217
under point reflection, 228
under rotation about a fixed point, 239
under translation, 234
Angle-side-angle triangle congruence, 162. See also ASA triangle congruence
Antecedent, 55
Apothem
of circle, 547
of polygon, 567
Arc(s).
of circle, 537
congruent, 538
degree measure of, 537-538
intercepted, 537
major, 537
minor, 537
types of, 537
Arc addition postulate, 539
Archimedes, 535
Area of a polygon, 409-410
Argument, valid, 75, 109
Arrowheads, 2
ASA triangle congruence, 162. See also Angle-side-angle triangle congruence
Associative property
of addition, 4-5
of multiplication, 5
Axiom, 93, 109-110
Axis of symmetry, 219, 625

## B

Base(s)
of cone, 456
of cylinder, 453
of polyhedron, 440
of rectangle, 389
of regular pyramid, 450
of trapezoid, 402
Base angles
of isosceles trapezoid, 403
of isosceles triangle, 25
lower, 404
upper, 404
Basic constructions, 196-202
Baudhāyana, 474
Betweenness, 8

Biconditional(s), 69-73
applications of, 70-73
definitions as, 97-99
Bisector
of angle, 20, 103, 137
of line segment, 12
Bolyai, János, 379
Boole, George, 34
Boolean algebra, 34

## C

Cartesian coordinates, 209
Cavalieri, Bonaventura, 419
Cavalieri's Principle, 419
Center
of circle, 460, 536
chords equidistant from, 546-550
of regular polygon, 450
of sphere, 459
Center-radius equation of a circle, 582
Centimeter, cubic, 446
Central angle of circle, 536-537
Centroid, 506
Chord(s)
angles formed by, 567-568
of circle, 543
equidistant from center of circle, 546-550
segments formed by two intersecting, 575
Circle(s), 460
apothem of, 547
arc of, 537
center of, 460, 536
central angle of, 536-537
chords of, 543, 546-550
circumference of, 535
concentric, 612
congruent, 538
in coordinate plane, 581-584
defined, 536
diameter of, 543
exterior of, 536
inscribed angles of, 552-553
interior of, 536
polygons circumscribed about, 563-564
polygons inscribed in, 550-551
radius of, 536
secant of, 558
squaring of, 604
tangent to, 558
Circular cylinder, 453
surface area of, 454
volume of, 454
Circumcenter, 194
Circumference of circle, 535
Circumscribed circle, 550
Circumscribed polygon, 563
Closed sentence, 37
Closure property
of addition, 4
of multiplication, 4
Collinear set of points, 7
Collinearity, preservation of
under line reflection, 217
under translation, 234
Common external tangent, 560
Common internal tangent, 559

Common tangent, 559-561
Commutative property
of addition, 4
of multiplication, 4
Compass, 196, 604
Complementary angles, 145
Complements, 145
Composition of transformations, 243, 251-252
Compound locus, 614
Compound sentence(s), 42, 53
conjunctions as, 42-46
Compound statement(s), 42,53
conditionals as, 4253-57
disjunctions as, 48-50
Concave polygon, 368
Concentric circles, 612
Conclusion(s), 55
drawing, 80-83
Concurrence, 193
of altitudes of triangle, 317-320
of angle bisectors of triangle, 364-365
Conditional(s), 53-57, 60,335
contrapositive of, 64-65
converse of, 63-64
false, 63, 65
hidden, 55-57, 98
inverse of, 61-62
parts of, 55
as they relate to proof, 138-139
true, 63, 64
truth value of, 54-55
Cone(s), 456-458
altitude of, 456
base of, 456
frustum of, 459
height of, 456
right circular, 456
slant height of, 456
surface area of, 457-458
volume of, 457-458
Congruence
equivalence relations of, 156-157
of triangles, 134, 174-203
Congruent angles, 19, 103
Congruent arcs, 538
Congruent circles, 538
Congruent polygons, 154-155
corresponding parts of, 155
Congruent segments, 9
Congruent triangles, $155-156$
in proving line segments congruent and angles congruent, 178-179
using two pairs of, 186
Conjecture, 95, 97
Conjunct, 42
Conjunction, 42-46
Consecutive angles
of polygon, 368
of quadrilateral, 380
Consecutive sides of quadrilateral, 380
Consecutive vertices
of polygon, 368
of quadrilateral, 380
Consequent, 55
Constant of dilation, $k, 501$
Constant of proportionality, 486

Constructions
of angle bisector, 199
of congruent angles, 197
of congruent segments, 196-197
locus in, 609-611
of midpoint, 198
of parallel lines, 605-606
of perpendicular bisector, 198
of perpendicular through a point not on the line, 201
of perpendicular through a point on the line, 200
Contraction, 496
Contrapositive, 64-65
Converse, 63-64
of Isosceles Triangle Theorem, 357-360
of Pythagorean Theorem, 516-517
Converse statement, 335
Convex polygon, 368
Coordinate geometry, 209, 290
equidistant lines in, 619-622
points at fixed distance in, 616-618
Coordinate of point, 4,7
Coordinate plane, 210
circles in, 581-584
dilations in, 247-248
line reflections in, 222-225
locating points in, 210-211
parallel lines in, 342-344
point reflections in, 227-231
rotations in, 238-242
secants in, 588-591
tangents in, 588
translations in, 232-235
preservation of angle measure, 232
preservation of collinearity, 232
preservation of distance, 232
preservation of midpoint, 232
Coordinate proof, 313-314
of general theorems, 313
for special cases, 313
Coordinates, 210
of point in plane, 210-213
rectangular, 211
Coplanar lines, 329
Corollary, 182
Corresponding angles, 155,330
Corresponding parts of congruent polygons, 155
Corresponding sides, 155
Counterexample, 95
Cube, 445
duplication of, 604
Cubic centimeter, 446
Cylinder, 453-455
altitude of, 453
bases of, 453
circular, 453
height of, 453
lateral surface of, 453
right circular, 453

## D

Decagon, 367
Deductive reasoning, 97, 100-103, 150-151
Definition(s), 7, 93
as biconditionals, 97-99
qualities of good, 7

Definition(s) cont.
using, in proofs, 141-142
writing as conditionals, 98
Degree measure
of angle, 16
of arc, 537-538
DeMorgan, Augustus, 34
DeMorgan's Laws, 34
Descartes, René, 209, 290
Detachment, law of, 75, 101, 105
Diagonal
of polygon, 368
of quadrilateral, 380
Diagram(s)
tree, 42
using, in geometry, 26-27
Diameter, 543
Dihedral angle, 424
Dilation(s), 495-499
in coordinate plane, 247-248
defined, 247
preservation of angle measure under, 496-497
preservation of collinearity under, 498
preservation of midpoint under, 497
Direct isometry, 252-253
Direct proof, 105-106
Disjunct, 48
Disjunction, 48-50, 76
Disjunctive inference, law of, 76-78
Distance,
between two parallel lines, 383
between two planes, 437
between two points, $7-8$
from a point to a line, 20
preservation of, under glide reflection, 244
preservation of, under point reflection, 228
preservation of, under rotation about a fixed point, 239
preservation of, under translation, 234
Distance formula, 521-522, 522
Distance postulate, 136
Distributive property, 5
Divide and average method, 174
Division postulate, 124, 270-271
$D_{k}, 247,496$
Domain, 36, 250

## E

Edge of polyhedron, 440
Elements (Euclid), 1, 262, 474
Endpoint, of ray, 15
Enlargement, 496
Epicureans, 262
Equality
reflexive property of, 110-111
symmetric property of, 111
transitive property of, 111, 263-264
Equation(s)
of line, 295-299
solving, with biconditionals, 70-71
Equiangular triangle, 25
Equidistant, 191
Equidistant lines in coordinate geometry, 619-622
Equilateral triangle(s), 24, 25, 181-183
properties of, 183

Equivalence relation, 111
of congruence, 156-157
of similarity, 487-488
Eratosthenes, 1
Euclid, 1, 93, 134, 262, 328, 379, 535
parallel postulate of, 328
Euclid Freed of Every Flaw, 379
Exclusive or, 51
Exterior angle(s), 330
alternate, 330
of polygon, 276-277, 369-371
of triangle, 277-279
Exterior angle inequality theorem, 277
Exterior angle theorem, 349
Exterior of angle, 16
Exterior of circle, 536
External segment, 576
Extremes, 476

## F

Face(s) of polyhedron, 440
Fermat, Pierre de, 209
Fixed point, 214
Foot of altitude, 525
Foot of perpendicular, 20
Formula(s)
angles,
central, 555
inscribed, 555
formed by tangents, chords, and secants, 571-572
of polygons, 369
area of a rectangle, 409
circle, 582
distance, 522
Heron's, 174
lateral area,
of cone, 457, 467
of cylinder, 454,467
of prism, 442, 467
of pyramid, 467
midpoint, 304
point-slope, 297
segments formed by tangents, chords, and secants, 579
slope, 292, 297
surface area,
of cone, 457
of cylinder, 454,467
of prism, 442, 467
of pyramid, 467
of sphere, 462,467
volume,
of cone, 457,467
of cylinder, 454,467
of prism, 446, 467
of pyramid, 449, 467
of sphere, 462, 467
Foundations of Geometry (Hilbert), 93
45-45-degree right triangle, 517-518
Frustum of cone, 459
Function(s)
defined, 250
transformations as, 250-254

## G

Galileo, 419
Generalization, 94
General quadrilateral, 380

Geometric constructions, 196. See also Constuctions
Geometric inequalities, 262-285
basic inequality postulates, 263-265
inequalities involving lengths of the sides of a triangle, 273-274
inequalities involving sides and angles of a triangle, 281-284
inequality involving an exterior angle of a triangle, 276-279
inequality postulates involving addition and subtraction, 267-268
inequality postulates involving multiplication and division, 270-271
Geometric mean, 478
Geometry
analytic, 209
coordinate, 209, 290
deductive reasoning, 100-103
defined, 2
definitions as biconditionals, $97-99$
inductive reasoning, 94-97
non-Euclidean, 376
spherical, 32
proving statements in, 93-130
addition and subtraction postulates, 118-122
direct proofs, 105-108
indirect proofs, 105-108
multiplication and division postulates, 124-126
postulates, theorems, and proof, 109-115
substitution postulate, 115-117
solid, 420
using diagrams in, 26-27
using logic to form proof, 100-103
Glide reflection, 243-245
Graphing polygons, 212-213
Graphs, 4
Great circle of sphere, 460,461

## H

Half-line, 14-15
Heath, Thomas L., 1
Height. See also Altitude
of cone, 456
of cylinder, 453
of prism, 440
of pyramid, 449
Heron of Alexandria, 174
Heron's formula, 174
Hexagon, 367
Hidden conditional, 55-57, 98
Hilbert, David, 93
HL triangle congruence theorem, 362-365
Hypotenuse, 26
Hypotenuse-leg triangle congruence theorem, 362-365. See also HL triangle congruence theorem
Hypothesis, 55

## I

Identity
additive, 5
multiplicative, 5
Identity property, 5
If $p$ then $q, 53$
Image, 214, 215
Incenter, 364
Included angle, 24
Included side, 24
Inclusive or, 51
Incomplete sentences, 35
Indirect proof, 105-108, 283, 309, 331, 336, 425, 429, 431, 434, 436, 559
Inductive reasoning, 94-97
Inequality
geometric, 262-285
involving exterior angle of triangle, 276-279
involving lengths of sides of triangle, 273-274
involving sides and angles of triangle, 281-284
transitive property of, 264
Inequality postulate(s), 263, 265
involving addition and subtraction, 267-268
involving multiplication and division, 270-271
relating whole quantity and its parts, 263
transitive property, 263-264
Inscribed angle
of circle, 552-553
measures of, 552-555
Inscribed circle, 563
Inscribed polygon, 550
Intercepted arc, 537
Interior angle(s), 330
alternate, 330
on the same side of the transversal, 330
of polygon, 368-369
Interior of angle, 16
Interior of circle, 536
Intersecting lines, equidistant from two, 621-622
Intersection of perpendicular bisectors of sides of triangle, 193-195
Inverse of a conditional, 61
Inverse property, 5
Inverses, 61-62
additive, 5
multiplicative, 5
Isometry, 244
direct, 252-253
opposite, 253
Isosceles quadrilateral, 379
Isosceles trapezoid(s)
base angles of, 403, 404
properties of, 403
proving that quadrilateral is, 403-407
Isosceles triangle(s), 24, 25, 181-183, 451
base angle of, 25
parts of, 25
vertex angle of, 25
Isosceles triangle theorem, 181
converse of, 357-360

## L

Lateral area of prism, 442
Lateral edges of prism, 440, 441

Lateral sides of prism, 440
Lateral surface of cylinder, 453
Law(s)
DeMorgan's, 34
of Detachment, 75, 101, 105
of Disjunctive Inference, 76
of logic, 35, 74-78
Leg(s)
proving right triangles congruent by hypotenuse, 362-365
of right triangle, 26
of trapezoid, 402
Leibniz, Gottfried, 34, 290
Length of line segment, 9
Line(s), 1, 2, 7, 420-422
coplanar, 329
equation of, 295-299
equidistant, in coordinate geometry, 619-622
number, 3-4
order of points on, 8
parallel, 421, 605-606
in coordinate plate, 342-344
parallel to a plane, 433
perpendicular, 20, 100, 149
methods of proving, 193
planes and, 423-431
slopes of, 307-310
points equidistant from point and, 624-629
postulates of, 135-138
skew, 421-422
slope of, 291-294
straight, 1,2
Linear pair of angles, 148
Line of reflection, 214
Line reflection(s), 214-220
in coordinate plane, 222-225
preservation of angle measure under, 217
preservation of collinearity under, 217
preservation of distance under, 215
preservation of midpoint under, 217
Line segment(s), 9
addition of, 12-13
associated with triangles, 175-177
bisector of, 12
congruent, 9
construction of congruent segment, 196-197
construction of perpendicular bisector and midpoint, 198
divided proportionally, 482
formed by intersecting secants, 576-578
formed by tangent intersecting secant, 575-576
formed by two intersecting chords, 575
length or measure of, 9
on a line, projection of, 510
methods of proving perpendicular, 193
midpoint of, 11-12, 300-305
perpendicular bisector of, 191-195
postulates of, 135-138
proportions involving, 480-484
subtraction of, 12-13
tangent, 561-563
using congruent triangles to prove congruent, 178-179
Line symmetry, 218-220
Lobachevsky, Nicolai, 379
Locus, 604-630
compound, 614
discovering, 610-611
equidistant from two intersecting lines, 613
equidistant from two parallel lines, 614
equidistant from two points, 613
fixed distance from line, 614
fixed distance from point, 614
meaning of, 609-611
Logic, 34-92
biconditionals in, 69-73
conditionals in, 53-57
conjunctions in, 42-46
contrapositive in, 64-65
converse in, 63-64
defined, 35
disjunctions in, 48-50
drawing conclusions in, 80-83
equivalents in, 65-67
in forming geometry proof, 100-103
inverse in, 61-62
law(s) of, 35, 74-78
detachment, 75
disjunctive inference, 76-78
negations in, 38
nonmathematical sentences and phrases in, 35-36
open sentences in, 36-37
sentences and their truth values in, 35
statements and symbols in, 37
symbols in, 38-39
two uses of the word or, 51
Logical equivalents, 65-67
Lower base angles of isosceles trapezoid, 404

## M

Major arc, 537
Mathematical sentences, 35
Mean(s), 476
geometric, 478
Mean proportional, 478-479, 512
Measure
of angle, 16, 17, 552-555
formed by tangent intersecting a secant, 568
formed by two intersecting chords, 568
formed by two intersecting secants, 569
formed by two intersecting tangents, 569
of arc, 537
of central angle of a circle, 537
of inscribed angle of a circle, 553
of segments formed by
intersecting chords, 575
intersecting secants, 576-577
tangent intersecting a secant, 575-576
Median
of trapezoid, 405

Median cont.
of triangle, 175-176
Metrica (Heron of Alexandria), 174
Midpoint
of line segment, 11-12, 300-305
preservation of
under dilation, 497
under glide reflection, 244
under line reflection, 217
under point reflection, 228
under rotation about a fixed point, 239
under translation, 234
Midpoint formula, 304
Midsegment, 346-347, 480
Midsegment theorem, 480
Minor arc, 537
Multiplication
associative property of, 5
closure property of, 4
commutative property of, 4
distributive property of, 5
inequality postulates involving, 270-271
Multiplication postulate, 124, 270-271
Multiplication property of zero, 5
Multiplicative identity, 5
Multiplicative inverses, 5

## N

Negation, 38
Negative rotation, 239
Negative slope, 293
Newton, Isaac, 290
$n$-gon, 367
Noncollinear set of points, 7
Nonadjacent interior angle, 277
Nonmathematical sentences, phrases and, 35-36
No slope, 293
Number(s)
properties of system, 110
rational, 3
real, 3
Number line, 3-4
distance between two points on the real, 8
Numerical operation, 4

Obtuse angle, 17
Obtuse triangle, 25
Octagon, 367
One-to-one correspondence, 154
Open sentence, 36-37
Opposite angle, 24
Opposite isometry, 253
Opposite rays, 15
Opposite side, 24
Or
exclusive, 51
inclusive, 51
Order of points on a line, 8-9
Ordered pair, 210
Ordinate, 210
Orientation, 252-254
Origin, 210
Orthocenter, 319

## P

Parabola, turning point of, 626
Paragraph proof, 101-102
Parallelepiped(s), 441, 442
rectangular, 442
volume of, 446
Parallel lines, 328-372, 421
constructing, 605-606
in coordinate plane, 342-344
defined, 329
equidistant from two, 621
methods of proving, 333
planes and, 433-439
properties of, 335-340
proving, 329-333
in space, 421
transversals and, 330-332
Parallelogram(s), 380-383
proof of quadrilateral as, 385-387
properties of, 383
Parallel planes, 438
Partition postulate, 118-119
Pentagon, 23, 367
Perpendicular bisector
construction of, 198
of line segment, 191-195
Perpendicular Bisector Concurrence Theorem, 193
Perpendicular lines, 20, 100, 149
construction of
through given point on line, 200
through point not on the given line, 201
methods of proving, 193
planes and, 423-431
slopes of, 307-310
Phrase(s), 35
nonmathematical sentences and, 35-36
Plane(s), 420-422. See also Coordinate plane
coordinates of point in, 210-213
defined, 2
distance between two, 437
naming of, 2
parallel, 438
symmetry, 464
Plane angle, 424
Playfair, John, 328
Playfair's postulate, 328
Point(s), 1, 2, 7, 420-422
collinear, 7
coordinate of, 4, 7
distance between two, 7-8
equidistant from two, 619-620
finding coordinates of, in plane, 211
fixed, 214
at fixed distance in coordinate geometry, 616-618
locating, in coordinate plane, 210-211
noncollinear set of, 7
order of, on a line, 8
points equidistant from line and, 624-629
projection of, on line, 510
Point reflection, 227-229
in coordinate plane, 229-231
properties of, 228-229

Point reflections in the coordinate plane, 247-248
preservation of angle measure under, 228
preservation of collinearity under, 228
preservation of distance under, 228
preservation of midpoint under, 228
Points equidistant from point and line, 624-629
Point-slope form of equation of line, 297
Point symmetry, 229
Polygon(s), 23, 367
areas of, 409-410
circumscribed about a circle, 563-564
concave, 368
convex, 368
diagonals of, 368
exterior angles of, 276-277, 369-371
graphing, 212-213
inscribed in circle, 550-551
interior angles of, 368-369
regular, 369
sides of, 23
similar, 486-488
Polyhedron(s), 440-442
bases of, 440
edges of, 440
faces of, 440
vertices of, 440
Positive rotation, 239
Positive slope, 292
Postulate(s), 93, 109-110
addition, 119-120, 267-268, 539
basic inequality, 263-265
of the coordinate plane, 210
division, 124, 270-271
Euclid's parallel, 328
first, in proving statements, 110-113
of lines, line segments, and angles, 135-138
multiplication, 124, 270-271
partition, 118-119
Playfair's 328
powers, 124
relating a whole quantity and its parts, 263
roots, 125-126
of similarity, 490
substitution, 115-117, 264
subtraction, 121-122, 267-268
trichotomy, 264-265
using, in proofs, 141-142
Powers postulate, 124
Preimage, 214,215
Premise, 55, 75
Prism, 440
altitude of, 440
height of, 440
lateral area of, 442
lateral edges of, 440, 441
lateral sides of, 440
right, 441
surface area of, 440-443
total surface area of, 442
volume of, 446-447

Projection
of point on a line, 510
of segment on a line, 510
Proof(s), 100
conditional statements as they relate to, 138-139
by contradiction, 105
direct, 105-106
indirect, 106-108, 283, 309, 331, 336, 425, 429, 431, 434, 436, 559
paragraph, 101-102
transformational, 308
two-column, 101
using postulates and definitions in, 141-142
Proportion(s), 476-477
extremes of, 476
involving line segments, 480-484
in right triangle, 510-513
Proportionality, constant of, 486
Proving triangles
congruent by
AAS, 352-355
ASA, 161-163
SAS, 158-159
SSS, 165
HL, 362-365
similar by, 489-494
AA~,489-494
SSS~, 489-494
SAS~, 489-494
Pyramid(s), 449
altitude of, 449
height of, 449
properties of regular, 450-451
regular, 449
slant height of, 449
surface area of, 449
vertex of, 449
volume of, 449
Pythagoras, 1,474
Pythagorean Theorem, 474, 515
converse of, 516-517
Pythagorean triple, 517

## Q

Quadrilateral(s), 23, 212, 379-412
consecutive angles of, 380
diagonal of, 380
general, 380
isosceles, 379
opposite angles of, 380
opposite sides of, 380
proof of
as isosceles trapezoid, 403-407
as parallelogram, 385-387
as rectangle, 390
as rhombus, 394-396
as square, 399-401
Quarter turn, 240

## R

Radius
of circle, 536
of sphere, 459
Range, 250

Ratio
defined, 475-476
of similitude, 486
Rational numbers, 3
Ray, 14-15
endpoint of, 15
Real number line, distance between two points on, 8
Real numbers, 3
properties of system, 4-5
Reasoning
deductive, 97, 100-103, 150-151
inductive, 94-97
Rectangle(s), 389-391
altitude of, 389
angles of, 389
base of, 389
diagonals of, 389
proof of quadrilateral as, 390
properties of, 390
Rectangular coordinates, 211
Rectangular parallelepiped, 442
Rectangular solid, 442-443
Reflection(s)
glide, 243-245
line, 214-220
line of, 214
in line $y=x, 224-225$
point, 227-231
in $x$-axis, 223
in $y$-axis, 222-223
Reflexive property, 156, 487
of equality, 110-111
Regular polygon(s), 369
center of, 450
Regular pyramid(s), 449
lateral faces of, 451
lateral sides of, 451
properties of, 450-451
Remote interior angle, 277
Replacement set, 36
Rhombus, 393-396
diagonals of, 394
proof of quadrilateral as, 394-396
properties of, 394
Riemann, Georg, 379
Right angle, 17
Right circular cone, 456
Right circular conical surface, 456
Right circular cylinder, 453
Right prism, 441
Right triangle(s), 25, 26,515
45-45 degree, 517-518
proportions in, 510-513
proving congruent by hypotenuse, leg, 362-365
Pythagorean theorem and, 515-517
30-60 degree, 518
$r_{k}, 217$
$R_{o}, 230$
Roots postulate, 125-126
Rotation
in coordinate plane, 238-239, 240-242
defined, 238
negative, 239
positive, 239
preservation of distance under, about a fixed point, 239

Rotational symmetry, 239-242
$R_{P}, 229$
$R_{P, d}, 239$
$r_{x \text {-axis }}, 223$
$r_{y-\text {-xis }}, 2224$

## S

Saccheri, Girolamo, 379
SAS triangle congruence, 158. See also Side-angle-side triangle congruence
SAS similarity theorem (SAS~), 491-492. See also Side-angleside similarity theorem
Scalene triangle, 24, 25, 98
Secant(s)
angle formed by two intersecting, 568-571
of circle, 558
in coordinate plane, 588-591
external segment of, 576
segment formed by intersecting, 576-578
segment formed by tangent intersecting, 575-576
Segment. See Line segments
Semicircle, 537
Semiperimeter, 174
Sentence
closed, 37
compound, 42,53
incomplete, 35
mathematical, 35
nonmathematical, 35-36
open, 36-37
truth value of, 35
Set, 2, 7
noncollinear, of points, 7
replacement, 36
solution, 36
truth, 36
Side(s)
adjacent, 380
of angle, 15
classifying triangles according to, 24-25
consecutive, 380
corresponding, 155
inequalities involving, for triangle, 281-284
of polygon, 23
Side-angle-side similarity theorem, 491-492. See also SAS similarity theorem
Side-angle-side triangle congruence, 158, See also SAS triangle congruence
Side-side-side similarity theorem, 491. See also SSS similarity theorem
Side-side-side triangle congruence, 165 . See also SSS triangle congruence
Side-side-side triangle similarity, 491. See also SSS triangle similarity
Similar polygons, 486-488

Similarity
equivalence relation of, 487-488
postulate of, 490
Similitude, ratio of, 486
Skew lines, 421-422
Slant height of cone, 456
of pyramid, 449
Slope
of line, 291-294
negative, 293
of perpendicular lines, 307-310
positive, 292
undefined, 293
zero, 293
Slope-intercept form of an equation, 298
Solid geometry, 420
Solid, rectangular, 442-443
Solution set, 36
Sphere, 459-463
center of, 459
great circle of, 460
radius of, 459
surface area of, 462-463
volume of, 462-463
Square(s), 399-401
properties of, 399
proving that quadrilateral is, 399-401
Squaring of circle, 604
SSS similarity theorem (SSS~), 491. See also, Side-side-side similarity theorem
SSS triangle congruence, 165. See also Side-side-side triangle congruence
Statement(s)
compound, 42, 53
negation of, 38
postulates used in proving, 110-113
symbols and, 37
Straight angle, 16, 17
Straight line, 1,2
Straightedge, 196, 604
Substitution postulate, 115-117, 264, 267-268
Subtraction
of angles, 20-21
inequality postulates involving, 267-268
of line segments, 12-13
Subtraction postulate, 121-122
Sum of two angles, 20
Supplementary angles, 145-146, 148-149
Supplements, 146
Surface, 1
Surface area
of circular cylinder, 454
of cone, 457-458
of prism, 440-443
of pyramid, 449
of sphere, 462-463
Symbols
logic, 38-39
statements and, 37
Symmetric property, 156, 487
of equality, 111

Symmetry
axis of, 219,625
line, 218-220
line of, 219
point, 229
rotational, 239-242
translational, 235
Symmetry plane, 464

## T

$T_{a, b}, 235$
Tangent(s)
angles formed by two intersecting, 567-568, 568-571
to a circle, 558
common, 559-561
common external, 560
in coordinate plane, 588
externally, 560
internally, 560
segments formed by, intersecting a secant, 575-576
Tangent segment, 561-563
Tangent to circle, 558
Terms, undefined, 1,2
Tessellation, 259
Thales, 1
Theorem(s), 110
defined, 141
Exterior Angle, 349
Exterior Angle Inequality, 277
Hypotenuse-Leg Triangle Congruence, 362-365
involving pairs of angles, 146-148
Isosceles Triangle, 181
converse of, 357-360
Midsegment, 480
Perpendicular Bisector Concurrence, 193
proving, about angles, 144-145
Pythagorean, 474,515
converse of, 515-517
Triangle Inequality, 273
Triangle Similarity, 490
30-60-degree right triangle, 518
Total surface area of prism, 442
Transformation(s), 215
composition of, 251-252
as functions, 250-254
dilation, 247-248, 495-499
glide reflection, 243-245
line reflection, 214-220
point reflection, 227-231
rotation, 238-239, 240-242
translation, 232-235
Transformational proof, 308
Transitive property, 156, 157, 263-264, 487,488
of equality, 111
of inequality, 264
Translation(s)
of $a$ units in the horizontal direction and $b$ units in the vertical direction, 233
in coordinate plane, 232-235
preservation of angle measure, 232
preservation of collinearity, 232
preservation of distance, 232
preservation of midpoint, 232
defined, 232
Translational symmetry, 235
Transversal(s)
defined, 330
parallel lines and, 330-332
Trapezoid, 402-407
bases of, 402
legs of, 402
median of, 405
Tree diagram, 42
Triangle(s), 23
acute, 25, 105
altitude of, 175
angle bisector of, 176
centroid of, 506
classifying
according to angles, 25-26
according to sides, 24-25
concurrence of altitudes of, 317-320
concurrence of angle bisectors of, 364-365
concurrence of medians of, 506-509
congruence of, 134, 155-156, 174-203
defined, 23
equiangular, 25
equilateral, 24, 25, 181-183
exterior angles of, 277-279
included sides and included angles of, 24
inequalities
involving an exterior angle of, 276-279
involving lengths of sides of, 273-274
involving sides and angles of, 281-284
isosceles, 24, 25, 181-183, 451
line segments associated with, 175-177
median of, 175-176
obtuse, 25
opposite sides and opposite angles in, 24
proportional relations among segments related to, 502-505
proving congruent by
AAS, 352-355
ASA, 161-163
SAS, 158-159
SSS, 165
HL, 362-365
proving similar by
AA~, 489-494
SSS~, 489-494
SAS~, 489-494
right, $25,26,515$
45-45 degree, 517-518
proportions in, 510-513
Pythagorean theorem and, 515-517
30-60 degree, 518
scalene, 24, 25, 98
sum of measures of angles of, 347-350
Triangle Inequality Theorem, 273
Triangle Similarity Theorem, 490
Triangular prism, volume of, 447

Trichotomy postulate, 264-265
Trisection of angle, 604
Truth set, 36
Truth table, 42-44, 51
Truth value, $35,37,38$ for conditional $p \rightarrow q, 54-55$ sentences and, 35
al-Tusi, Nasir al-Din, 379
Two-column proof, 101

## U

Undefined term, 1,2
Upper base angles, 404
of isosceles trapezoid, 404

## V

Valid argument, 75,109
Value(s)
absolute, 7-8
truth, 35, 37, 38, 54-55

## Vertex

of angle, 15
of cone, 456
of pyramid, 449
Vertex angle of isosceles triangle, 25
Vertical angles, 149 as congruent, 150-151
Vertices
adjacent, 368, 380
consecutive, 380
of polyhedron, 440
Volume
of circular cylinder, 454
of cone, 457-458
of prism, 446-447
of pyramid, 449
of sphere, 462-463
of triangular prism, 447

## X

$x$-axis, reflection in, 223
$x$-coordinate, 210
$x$-intercept, 296
( $x, y$ ), 210

## Y

$y$-axis, reflection in, 222-223
$y$-coordinate, 210
$y$-intercept, 296

## $\mathbf{Z}$

Zero, multiplication property of, 5 Zero slope, 293

