

GRAPHING LINEAR FUNCTIONS AND RELATIONS

The Tiny Tot Day Care Center charges \$200 a week for children who stay at the center between 8:00 A.M. and 5:00 P.M. If a child is not picked up by 5:00 P.M., the center charges an additional \$4.00 per hour or any part of an hour.

Mr. Shubin often has to work late and is unable to pick up his daughter on time. For Mr. Shubin, the weekly cost of day care is a *function* of time; that is, his total cost depends on the time he arrives at the center.

This function for the daily cost of day care can be expressed in terms of an equation in two variables: $y = 30 + 4x$. The variable x represents the total time, in hours, after 5:00 P.M., that a child remains at the center, and the variable y represents the cost, in dollars, of day care for the day.

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9-1 SETS, RELATIONS, AND FUNCTIONS

Set-Builder Notation

In Chapters 1 and 2, we used **roster form** to describe sets. In roster form, the elements of a set are enclosed by braces and listed once. Repeated elements are not allowed. For example,

$$\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$$

is the set of integers, in roster form.

A second way to specify a set is to use **set-builder notation**. Set-builder notation is a mathematically concise way of describing a set without listing the elements of the set. For instance, using set-builder notation, the set of counting numbers from 1 to 100 is:

$$\{x \mid x \text{ is an integer and } 1 \leq x \leq 100\}$$

This reads as “the set of all x such that x is an integer and x is at least 1 and at most 100.” The vertical bar “|” represents the phrase “such that,” and the description to the right of the bar is the rule which defines the set.

Here are some other examples of set-builder notation:

- $\{x \mid x \text{ is an integer and } x > 6\} = \{7, 8, 9, 10, \dots\}$ = the set of integers greater than 6
- $\{x \mid -1 \leq x \leq 3\}$ = any real number in the interval $[-1, 3]$
- $\{2n + 1 \mid n \text{ is a whole number}\} = \{2(0) + 1, 2(1) + 1, 2(2) + 1, 2(3) + 1, \dots\}$
 $= \{1, 3, 5, 7, \dots\}$
 = the set of odd whole numbers

Frequently used with set-builder notation is the symbol \in , which means “is an element of.” This symbol is used to indicate that an element is a member of a set. The symbol \notin means “is not an element of,” and is used to indicate that an element is not a member of a set. For instance,

$$2 \in \{x \mid -1 \leq x \leq 3\} \quad \text{since 2 is between } -1 \text{ and } 3.$$

$$-2 \notin \{x \mid -1 \leq x \leq 3\} \quad \text{since } -2 \text{ is not between } -1 \text{ and } 3.$$

EXAMPLE 1

List the elements of each set or indicate that the set is the empty set.

- | | |
|--|--|
| a. $A = \{x \mid x \in \text{of the set of natural numbers and } x < 0\}$ | b. $B = \{2n \mid n \in \text{of the set of whole numbers}\}$ |
| c. $C = \{x \mid x + 3 = 5\}$ | d. $D = \{m \mid m \text{ is a multiple of 5 and } m < -25\}$ |

Solution a. Since there are no natural numbers less than 0,

$$A = \emptyset$$

c. C represents the solution set of the equation $x + 3 = 5$. Therefore,

$$C = \{2\}$$

$$\begin{aligned} \mathbf{b. } B &= \{2(0), 2(1), 2(2), 2(3), \dots\} \\ &= \{0, 2, 4, 6, \dots\} \end{aligned}$$

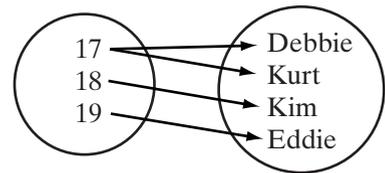
d. D consists of the set of multiples of 5 that are less than -25 .

$$D = \{\dots, -45, -40, -35, -30\}$$

Answers a. $A = \emptyset$ b. $B = \{0, 2, 4, 6, \dots\}$ c. $C = \{2\}$ d. $D = \{\dots, -45, -40, -35, -30\}$

Relations That are Finite Sets

There are many instances in which one set of information is related to another. For example, we may identify the persons of a group who are 17, 18, or 19 to determine who is old enough to vote. This information can be shown in a diagram such as the one at the right, or as a set of ordered pairs.



$$\{(17, \text{Debbie}), (17, \text{Kurt}), (18, \text{Kim}), (19, \text{Eddie})\}$$

DEFINITION

A **relation** is a set of ordered pairs.

The **domain of a relation** is the set of all first elements of the ordered pairs. For example, in the relation shown above, the domain is $\{17, 18, 19\}$, the set of ages.

The **range of a relation** is the set of all second elements of the ordered pairs. In the relation above, the range is $\{\text{Debbie}, \text{Kurt}, \text{Kim}, \text{Eddie}\}$, the set of people.

Let us consider the relation “is greater than,” using the set $\{1, 2, 3, 4\}$ as both the domain and the range. Let (x, y) be an ordered pair of the relation. Then the relation can be shown in any of the following ways.

1. A Rule

The relation can be described by the inequality $x > y$.

2. Set of Ordered Pairs

The relation can be listed in set notation as shown below:

$$\{(2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3)\}$$

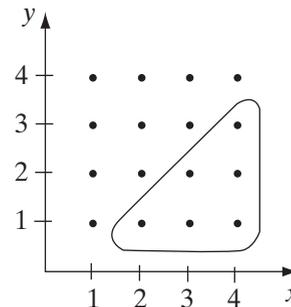
3. *Table of Values*

The ordered pairs shown in 2 can be displayed in a table.

x	2	3	3	4	4	4
y	1	1	2	1	2	3

4. *Graph*

In the coordinate plane, the domain is a subset of the numbers on the x -axis and the range is a subset of the numbers on the y -axis. The points that correspond to the ordered pairs of numbers from the domain and range are shown. Of the 16 ordered pairs shown, only six are enclosed to indicate the relation $x > y$.



Relations That are Infinite Sets

Charita wants to enclose a rectangular garden. How much fencing will she need if the width of the garden is to be 5 feet? The answer to this question depends on the length of the garden. Therefore, we say that the length of the garden, x , and the amount of fencing, y , form a set of ordered pairs or a relation. The formula for the perimeter of a rectangle can be used to express y in terms of x :

$$P = 2l + 2w$$

$$y = 2x + 2(5)$$

$$y = 2x + 10$$

Some possible values for the amount of fencing can also be shown in a table. As the length of the garden changes, the amount of fencing changes, as the following table indicates.

x	$2x + 10$	y	(x, y)
1	$2(1) + 10$	12	(1, 12)
3	$2(3) + 10$	16	(3, 16)
4.5	$2(4.5) + 10$	19	(4.5, 19)
7.2	$2(7.2) + 10$	24.4	(7.2, 24.4)
9.1	$2(9.1) + 10$	28.2	(9.1, 28.2)

Each solution to $y = 2x + 10$ is a pair of numbers. In writing each pair, we place the value of x first and the value of y second. For example, when $x = 1$, $y = 12$, $(1, 12)$ is a solution. When $x = 3$, $y = 16$, $(3, 16)$ is another solution. It is not possible to list all ordered pairs in the solution set of the equation because the solution set is infinite. However, it is possible to determine whether a given ordered pair is a member of the solution set. Replace x with the first element of the pair (the x -coordinate) and y with the second element of the pair (the y -coordinate). If the result is a true statement, the ordered pair is a solution of the equation.

- $(1.5, 13) \in \{(x, y) \mid y = 2x + 10\}$ because $13 = 2(1.5) + 10$ is true.
- $(4, 14) \notin \{(x, y) \mid y = 2x + 10\}$ because $14 = 2(4) + 10$ is false.

Since in the equation $y = 2x + 10$, x represents the length of a garden, only positive numbers are acceptable replacements for x . Therefore, the domain of this relation is the set of positive real numbers. For every positive number x , $2x + 10$ will also be a positive number. Therefore, the range of this relation is the set of positive real numbers. Although a solution of $y = 2x + 10$ is $(-2, 6)$, it is not possible for -2 to be the length of a garden; $(-2, 6)$ is not a pair of the relation.

When we choose a positive real number to be the value of x , there is one and only one value of y that makes the equation true. Therefore, $y = 2x + 10$ is a special kind of relation called a function.

DEFINITION

A **function** is a relation in which no two ordered pairs have the same first element.

or

A **function** is a relation in which every element of the domain is paired with one and only one element of the range.

Since a function is a special kind of relation, the **domain of a function** is the set of all first elements of the ordered pairs of the function. Similarly, the **range of a function** is the set of all second elements of the ordered pairs of the function. The notation $f(a) = b$ or “ f of a equals b ,” signifies that the value of the function, f , at a is equal to b .

The **independent variable** is the variable that represents the first element of an ordered pair. The domain of a function is the set of all values that the independent variable is allowed to take. The **dependent variable** is the variable that represents the second element of an ordered pair. The range of a function is the set of all values that the dependent variable is allowed to take.

EXAMPLE 2

Find five members of the solution set of the sentence $3x + y = 7$.

Solution*How to Proceed*

- (1) Transform the equation into an equivalent equation with y alone as one member:
- (2) Choose any five values for x . Since no replacement set is given, any real numbers can be used:
- (3) For each selected value of x , determine y :

$$\begin{array}{r} 3x + y = 7 \\ -3x \quad -3x \\ \hline y = -3x + 7 \end{array}$$

x	$-3x + 7$	y
-2	$-3(-2) + 7$	13
0	$-3(0) + 7$	7
$\frac{1}{3}$	$-3(\frac{1}{3}) + 7$	6
3	$-3(3) + 7$	-2
5	$-3(5) + 7$	-8

Answer $(-2, 13), (0, 7), (\frac{1}{3}, 6), (3, -2), (5, -8)$ ■

Note that many other solutions are also possible, and for every real number x , one and only one real number y will make the equation $3x + y = 7$ true. Therefore, $3x + y = 7$ defines a function.

EXAMPLE 3

Determine whether each of the given ordered pairs is a solution of the inequality $y - 2x \geq 4$.

a. $(4, 0)$ b. $(-1, 2)$ c. $(0, 6)$ d. $(0, 7)$ **Solution**

a. $y - 2x \geq 4$

$0 - 2(4) \stackrel{?}{\geq} 4$

$0 - 8 \geq 4$ ✗

c. $y - 2x \geq 4$

$6 - 2(0) \stackrel{?}{\geq} 4$

$6 - 0 \geq 4$ ✓

b. $y - 2x \geq 4$

$2 - 2(-1) \stackrel{?}{\geq} 4$

$2 + 2 \geq 4$ ✓

d. $y - 2x \geq 4$

$7 - 2(0) \stackrel{?}{\geq} 4$

$7 - 0 \geq 4$ ✓

Answers a. Not a solution b. A solution c. A solution d. A solution ■

Note that for the inequality $y - 2x \geq 4$, the pairs $(0, 6)$ and $(0, 7)$ have the same first element. Therefore $y - 2x \geq 4$ is a relation but not a function.

EXAMPLE 4

The cost of renting a car for 1 day is \$64.00 plus \$0.25 per mile. Let x represent the number of miles the car was driven, and let y represent the rental cost, in dollars, for a day.

- a.** Write an equation for the rental cost of the car in terms of the number of miles driven.
- b.** Find the missing member of each of the following ordered pairs, which are elements of the solution set of the equation written in part **a**, and explain the meaning of the pair.

(1) $(155, ?)$

(2) $(?, 69)$

Solution **a.** Rental cost is \$64.00 plus \$0.25 times the number of miles.

$$y = 64.00 + 0.25x$$

b. (1) For the pair $(155, ?)$, $x = 155$ and y is to be determined. Then:

$$\begin{aligned} y &= 64.00 + 0.25x \\ &= 64.00 + 0.25(155) \\ &= 64.00 + 38.75 \\ &= 102.75 \end{aligned}$$

When the car was driven 155 miles, the rental cost was \$102.75.

(2) For the pair $(?, 69)$, $y = 69$ and x is to be determined. Then:

$$\begin{aligned} y &= 64 + 0.25x \\ 69 &= 64 + 0.25x \\ -64 &\quad -64 \\ \hline 5 &= \quad + 0.25x \\ \\ \frac{5}{0.25} &= \frac{0.25x}{0.25} \\ 20 &= x \end{aligned}$$

When the rental cost was \$69, the car was driven 20 miles.

Answers **a.** $y = 64.00 + 0.25x$

b. (1) $(155, 102.75)$ The car was driven 155 miles, and the cost was \$102.75.

(2) $(20, 69)$ The car was driven 20 miles, and the cost was \$69.00. ■

EXAMPLE 5

Determine whether or not each set is a relation. If it is a relation, determine whether or not it is a function.

- a. $\{x \mid 0 < x < 10\}$
- b. $\{(x, y) \mid y = 4 - x\}$
- c. $\{(a, b) \mid a = 2 \text{ and } b \text{ is a real number}\}$
- d. $\{(x, y) \mid y = x^2\}$

- Solution**
- a. The set is not a set of ordered pairs. It is not a relation. *Answer*
 - b. The set is a relation and a function. It is a set of ordered pairs in which every first element is paired with one and only one second element. *Answer*
 - c. The set is a relation that is not a function. The same first element is paired with every second element. *Answer*
 - d. The set is a relation and a function. Every first element is paired with one and only one second element, its square. *Answer* ■

EXAMPLE 6

Jane would like to construct a rectangular pool having an area of 102 square feet. If l represents the length of the pool and w its height, express the set of all ordered pairs, (l, w) , that represent the dimensions of the pool using set-builder notation.

- Solution** Since the area must be 102 square units, the variables l and w are related by the area formula: $A = lw$. Therefore, the set of ordered pairs representing the possible dimensions of the pool is:

$$\{(l, w) \mid 102 = lw, \text{ where } l \text{ and } w \text{ are both positive}\} \text{ *Answer* } \quad \text{■}$$

Note: Since the length and the width cannot be zero or negative, the rule must specify that the dimensions, l and w , of the pool are both positive.

EXERCISES**Writing About Mathematics**

1. A function is a set of ordered pairs in which no two different pairs have the same first element. In Example 4, we can say that the cost of renting a car is a function of the number of miles driven. Explain how this example illustrates the definition of a function.

2. Usually when a car rental company charges for the number of miles driven, the number of miles is expressed as a whole number. For example, if the car was driven 145.3 miles, the driver would be charged for 146 miles. In Example 4, what would be the domain of the function? (Recall that the domain is the set of numbers that can replace the variable x .)

Developing Skills

In 3–7, find the missing member in each ordered pair if the second member of the pair is twice the first member.

3. $(3, ?)$ 4. $(0, ?)$ 5. $(-2, ?)$ 6. $(?, 11)$ 7. $(?, -8)$

In 8–12, find the missing member in each ordered pair if the first member of the pair is 4 more than the second member.

8. $(?, 5)$ 9. $(?, \frac{1}{2})$ 10. $(?, 0)$ 11. $(9\frac{1}{4}, ?)$ 12. $(-8, ?)$

In 13–27, state whether each given ordered pair of numbers is a solution of the equation or inequality.

- | | | |
|---|------------------------------|--------------------------------|
| 13. $y = 5x; (3, 15)$ | 14. $y = 4x; (16, 4)$ | 15. $y = 3x + 1; (7, 22)$ |
| 16. $3x - 2y = 0; (3, 2)$ | 17. $y > 4x; (2, 10)$ | 18. $y < 2x + 3; (0, 2)$ |
| 19. $3y > 2x + 1; (4, 3)$ | 20. $2x + 3y \leq 9; (0, 3)$ | 21. $x + y = 8; (4, 5)$ |
| 22. $4x + 3y = 2; (\frac{1}{4}, \frac{1}{3})$ | 23. $3x = y + 4; (-7, -1)$ | 24. $x - 2y \leq 15; (1, -7)$ |
| 25. $y > 6x; (-1, -2)$ | 26. $3x < 4y; (5, 2)$ | 27. $5x - 2y \leq 19; (3, -2)$ |

In 28–31, state which sets of ordered pairs represent functions. If the set is not a function, explain why.

28. $\{(1, 2), (2, 3), (3, 4), (5, 6)\}$ 29. $\{(1, 2), (2, 1), (3, 4), (4, 3)\}$
 30. $\{(-5, -5), (-5, 5), (6, -6), (6, 6)\}$ 31. $\{(81, 9), (81, -9), (25, 5), (25, -5)\}$

In 32–35, find the range of each function when the domain is:

- a. $A = \{x \mid 10 < x < 13 \text{ and } x \in \text{of the set of integers}\}$ b. $B = \{6x \mid x \in \text{of the set of whole numbers}\}$

32. $y = 2x + 3$ 33. $y = \frac{x}{6}$ 34. $y = -x - 2$ 35. $y = x + 1$

Applying Skills

In 36–39, in each case: **a.** Write an equation or an inequality that expresses the relationship between x and y . **b.** Find two ordered pairs in the solution set of the equation that you wrote in part **a.** **c.** Is the relation determined by the equation or inequality a function?

36. The cost, y , of renting a bicycle is \$5.00 plus \$2.50 times the number of hours, x , that the bicycle is used. Assume that x can be a fractional part of an hour.
37. In an isosceles triangle whose perimeter is 54 centimeters, the length of the base in centimeters is x and the length of each leg in centimeters is y .

38. Jules has \$265 at the beginning of the month. What is the maximum amount, y , that he has left after 30 days if he spends at least x dollars a day?

39. At the beginning of the day, the water in the swimming pool was 2 feet deep. Throughout the day, water was added so that the depth increased by less than 0.4 foot per hour. What was the depth, y , of the water in the pool after being filled for x hours?

40. A fence to enclose a rectangular space along a river is to be constructed using 176 feet of fencing. The two sides perpendicular to the river have length x .

Length (x)	Width	Area (A)
1	$176 - 2(1)$	$1[176 - 2(1)]$ $= 174$
2	$176 - 2(2)$	$2[176 - 2(2)]$ $= 344$

a. Complete five more rows of the table shown on the right.

b. Is the area, A , a function of x ? If so, write the function and determine its domain.

41. (1) At the Riverside Amusement Park, rides are paid for with tokens purchased at a central booth. Some rides require two tokens, and others one token. Tomas bought 10 tokens and spent them all on x two-token rides and y one-token rides. On how many two-token and how many one-token rides did Tomas go?

(2) Minnie used 10 feet of fence to enclose three sides of a rectangular pen whose fourth side was the side of a garage. She used x feet of fence for each side perpendicular to the garage and y feet of fence for the side parallel to the garage. What were the dimensions of the pen that Minnie built?

a. Write an equation that can be used to solve both problems.

b. Write the six solutions for problem (1).

c. Which of the six solutions for problem (1) are *not* solutions for problem (2)?

d. Write two solutions for problem (2) that are *not* solutions for problem (1).

e. There are only six solutions for problem (1) but infinitely many solutions for problem (2). Explain why.

42. A cylindrical wooden base for a trophy has radius r and height h . The volume of the base will be 102 cubic inches. Using set-builder notation, describe the set of ordered pairs, (r, h) , that represent the possible dimensions of the base.

9-2 GRAPHING LINEAR FUNCTIONS USING THEIR SOLUTIONS

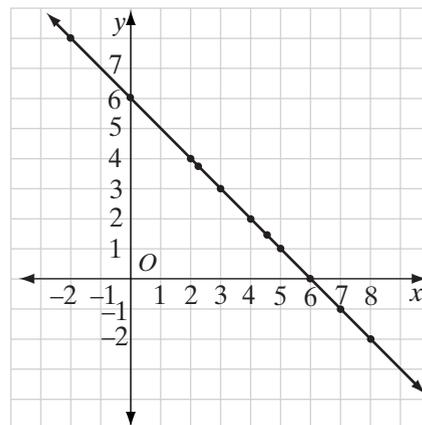
Think of two numbers that add up to 6. If x and y represent these numbers, then $x + y = 6$ is an equation showing that their sum is 6. If the replacement set for both x and y is the set of real numbers, we can find infinitely many solutions for the equation $x + y = 6$. Some of the solutions are shown in the following table.

x	8	7	6	5	4.5	4	3	2.2	2	0	-2
y	-2	-1	0	1	1.5	2	3	3.8	4	6	8

Each ordered pair of numbers, such as $(8, -2)$, locates a point in the coordinate plane. When all the points that have the coordinates associated with the number pairs in the table are located, the points appear to lie on a straight line, as shown at the right.

In fact, if the replacement set for both x and y is the set of real numbers, the following is true:

All of the points whose coordinates are solutions of $x + y = 6$ lie on this same straight line, and all of the points whose coordinates are not solutions of $x + y = 6$ do *not* lie on this line.



This line, which is the set of all the points and only the points whose coordinates make the equation $x + y = 6$ true, is called the **graph** of $x + y = 6$.

DEFINITION

A first-degree equation in **standard form** is written as

$$Ax + By = C$$

where A , B , and C are real numbers, with A and B not both 0.

We call an equation that can be written in the form $Ax + By = C$ a **linear equation** since its graph is a straight line that contains all points and only those points whose coordinates make the equation true. The replacement set for both variables is the set of real numbers unless otherwise indicated.

When we graph a linear equation, we can determine the line by plotting two points whose coordinates satisfy that equation. However, we usually plot a third point as a check on the first two. If the third point lies on the line determined by the first two points, we have probably made no error.

Procedure

To graph a linear equation by means of its solutions:

1. Transform the equation into an equivalent equation that is solved for y in terms of x .
2. Find three solutions of the equation by choosing values for x and finding corresponding values for y .
3. In the coordinate plane, graph the ordered pairs of numbers found in Step 2.
4. Draw the line that passes through the points graphed in Step 3.

EXAMPLE 1

Does the point $(2, -3)$ lie on the graph of $x - 2y = -4$?

Solution The point $(2, -3)$ lies on the graph of $x - 2y = -4$ if and only if it is a solution of $x - 2y = -4$.

$$\begin{aligned}x - 2y &= -4 \\2 - 2(-3) &\stackrel{?}{=} -4 \\2 + 6 &\stackrel{?}{=} -4 \\8 &\neq -4 \quad \times\end{aligned}$$

Answer Since $8 = -4$ is not true, the point $(2, -3)$ does not lie on the line $x - 2y = -4$. ■

EXAMPLE 2

What must be the value of d if $(d, 4)$ lies on line $3x + y = 10$?

Solution The coordinates $(d, 4)$ must satisfy $3x + y = 10$.
Let $x = d$ and $y = 4$. Then:

$$\begin{aligned}3d + 4 &= 10 \\3d &= 6 \\d &= 2\end{aligned}$$

Answer The value of d is 2. ■

EXAMPLE 3

a. Write the following verbal sentence as an equation:

The sum of twice the x -coordinate of a point and the y -coordinate of that point is 4.

b. Graph the equation written in part **a.**

Solution **a.** Let $x =$ the x -coordinate of the point, and $y =$ the y -coordinate of the point.
Then:

$$2x + y = 4 \quad \text{Answer}$$

b. *How to Proceed*

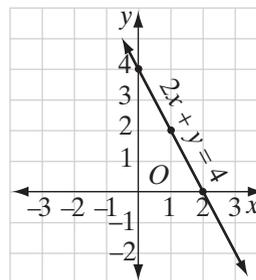
(1) Transform the equation into an equivalent equation that has y alone as one member:

$$\begin{aligned}2x + y &= 4 \\y &= -2x + 4\end{aligned}$$

- (2) Choose three values of x and find the corresponding values of y :

x	$-2x + 4$	y
0	$-2(0) + 4$	4
1	$-2(1) + 4$	2
2	$-2(2) + 4$	0

- (3) Plot the points that are associated with the three solutions:
- (4) Draw a line through the points that were plotted. Label the line with its equation:



To display the graph of a function on a calculator, follow the steps below.

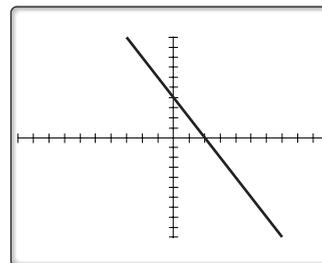
1. Solve the equation for y and enter it as Y_1 .

ENTER: **Y=** **(-)** 2 **X,T,θ,n** **+** 4 DISPLAY: **PLOT1** **PLOT2** **PLOT3**
ENTER **\Y₁** **=** **-2X+4**

2. Use the standard viewing window to view the graph. The standard viewing window is a 20 by 20 graph centered at the origin.

ENTER: **ZOOM** 6 **ENTER**

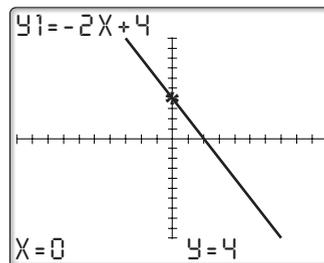
DISPLAY: **ZOOM** MEMORY
 1: ZBOX
 2: ZOOM IN
 3: ZOOM OUT
 4: ZDECIMAL
 5: ZSQUARE
 6: ZSTANDARD
 7: ZTRIG



3. Display the coordinates of points on the graph.

ENTER: **TRACE**

In the viewing window, the equation appears at the top left of the screen. The point at which the graph intersects the y -axis is marked with a star and the coordinates of that point appear at the bottom of the screen. Press the right and left arrow keys to move the star along the line to display other coordinates.



EXERCISES

Writing About Mathematics

- The points whose coordinates are $(3, 1)$, $(5, -1)$ and $(7, -3)$ all lie on the same line. What could be the coordinates of another point that lies on that line? Explain how you found your answer.
- Of the points $(0, 5)$, $(2, 4)$, $(3, 3)$, and $(6, 2)$, which one does not lie on the same line as the other three? Explain how you found your answer.

Developing Skills

In 3–5, state, in each case, whether the pair of values for x and y is a member of the solution set of the equation $2x - y = 6$.

3. $x = 4, y = 2$

4. $x = 0, y = 6$

5. $x = 4, y = -2$

For 6–9, state in each case whether the point whose coordinates are given is on the graph of the given equation.

6. $x + y = 7; (4, 3)$

7. $2y + x = 7; (1, 3)$

8. $3x - 2y = 8; (2, 1)$

9. $2y = 3x - 5; (-1, -4)$

In 10–13, find in each case the number or numbers that can replace k so that the resulting ordered number pair will be on the graph of the given equation.

10. $x + 2y = 5; (k, 2)$

11. $3x + 2y = 22; (k, 5)$

12. $x + 3y = 10; (13, k)$

13. $x - y = 0; (k, k)$

In 14–17, find in each case a value that can replace k so that the graph of the resulting equation will pass through the point whose coordinates are given.

14. $x + y = k$; (2, 5)

15. $x - y = k$; (5, -3)

16. $y - 2x = k$; (-2, -1)

17. $y = x + k$; (5, 0)

In 18–23, solve each equation for y in terms of x .

18. $3x + y = -1$

19. $4x - y = 6$

20. $2y = 6x$

21. $12x = \frac{3}{2}y$

22. $4x + 2y = 8$

23. $6x - 3y = 5$

In 24–26: **a.** Find the missing values of the variable needed to complete each table. **b.** Plot the points described by the pairs of values in each completed table; then, draw a line through the points.

24. $y = 4x$

25. $y = 3x + 1$

26. $x + 2y = 3$

x	y
0	?
1	?
2	?

x	y
-1	?
0	?
1	?

x	y
-1	?
2	?
5	?

In 27–50, graph each equation.

27. $y = x$

28. $y = 5x$

29. $y = -3x$

30. $y = -x$

31. $x = 2y - 3$

32. $x = \frac{1}{2}y + 1$

33. $y = x + 3$

34. $y = 2x - 1$

35. $y = 3x + 1$

36. $y = -2x + 4$

37. $x + y = 8$

38. $x - y = 5$

39. $y - x = 0$

40. $3x + y = 12$

41. $x - 2y = 0$

42. $y - 3x = -5$

43. $2x - y = 6$

44. $3x - y = -6$

45. $x + 3y = 12$

46. $2x + 3y = 6$

47. $3x - 2y = -4$

48. $x - 3y = 9$

49. $2x = y - 4$

50. $4x = 3y$

51. **a.** Through points (0, -2) and (4, 0), draw a straight line.**b.** Write the coordinates of two other points on the line drawn in part **a.**52. **a.** Through points (-2, 3) and (1, -3), draw a straight line.**b.** Does point (0, -1) lie on the line drawn in part **a.**?**Applying Skills**In 53–60: **a.** Write an equation that can be used to represent each sentence. **b.** Graph the equation.53. The length of a rectangle, y , is twice the width, x .54. The distance to school, y , is 2 miles more than the distance to the library, x .55. The cost of a loaf of bread, x , plus the cost of a pound of meat, y , is 6 dollars.

56. The difference between Tim's height, x , and Sarah's height, y , is 1 foot.
57. The measure of each of three sides of the trapezoid is x , the measure of the remaining side is y , and the perimeter is 6.
58. The measure of each of the legs of an isosceles triangle is x , the measure of the base is y , and the perimeter is 9.
59. Bob's age, y , is equal to five more than twice Alice's present age, x .
60. The sum of the number of miles that Paul ran, x , and the number of miles that Sue ran, y , is 30 miles.

9-3 GRAPHING A LINE PARALLEL TO AN AXIS

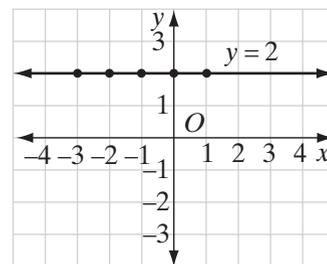
Lines Parallel to the x -Axis

An equation such as $y = 2$ can be graphed in the coordinate plane. Any pair of values whose y -coordinate is 2, no matter what the x -coordinate is, makes the equation $y = 2$ true. Therefore,

$$(-3, 2), (-2, 2), (-1, 2), (0, 2), (1, 2),$$

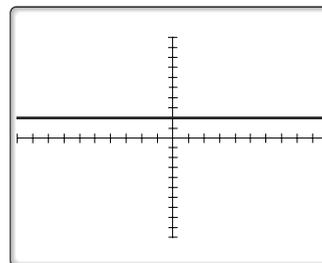
or any other pair $(a, 2)$ for all values of a , are points on the graph of $y = 2$. As shown at the right, the graph of $y = 2$ is a horizontal line parallel to the x -axis and 2 units above it.

The equation $y = 2$ defines a function and can be displayed on a graphing calculator. If there are other equations entered in the $Y=$ menu, press **CLEAR** before entering the new equation.



ENTER: **Y=** 2 **GRAPH**

DISPLAY:



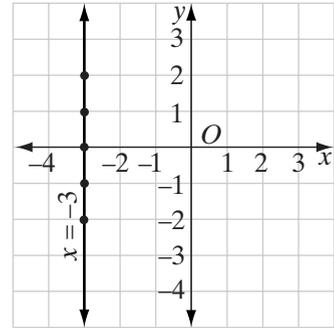
- The equation of a line parallel to the x -axis and b units from the x -axis is $y = b$. If b is positive, the line is above the x -axis, and if b is negative, the line is below the x -axis.

Lines Parallel to the y -Axis

An equation such as $x = -3$ can be graphed in the coordinate plane. Any pair of values whose x -coordinate is -3 , no matter what the y -coordinate is, makes the equation $x = -3$ true. Therefore,

$$(-3, -2), (-3, -1), (-3, 0), (-3, 1), (-3, 2),$$

or any other pair $(-3, b)$ for all values of b , are points on the graph of $x = -3$. As shown at the right, the graph of $x = -3$ is a vertical line parallel to the y -axis and 3 units to the left of it.



The equation $x = -3$ defines a relation that is not a function because it defines a set of ordered pairs but every ordered pair has the same first element. The method used to draw the graph of a function on a graphing calculator cannot be used to display this graph.

► **The equation of a line parallel to the y -axis and a units from the y -axis is $x = a$. If a is positive, the line is to the right of the y -axis, and if a is negative, the line is to the left of the x -axis.**

EXERCISES

Writing About Mathematics

1. Mike said that the equation of the x -axis is $x = 0$. Do you agree with Mike? Explain why or why not.
2. Does the line whose equation is $y = 4$ intersect the x -axis? Determine the coordinates of the point of intersection if one exists or explain why a point of intersection does not exist.

Developing Skills

In 3–17, draw the graph of each equation.

- | | | | | |
|-----------------------|------------------------|----------------|------------------------|---------------|
| 3. $x = 6$ | 4. $x = 4$ | 5. $x = 0$ | 6. $x = -3$ | 7. $x = -5$ |
| 8. $y = 4$ | 9. $y = 5$ | 10. $y = 0$ | 11. $y = -4$ | 12. $y = -7$ |
| 13. $x = \frac{1}{2}$ | 14. $y = 1\frac{1}{2}$ | 15. $y = -2.5$ | 16. $x = -\frac{3}{2}$ | 17. $y = 3.5$ |

18. Write an equation of the line that is parallel to the x -axis

- | | | |
|----------------------------|------------------------------|------------------------------|
| a. 1 unit above the axis. | b. 5 units above the axis. | c. 4 units below the axis. |
| d. 8 units below the axis. | e. 2.5 units above the axis. | f. 3.5 units below the axis. |

19. Write an equation of the line that is parallel to the y -axis
- a. 3 units to the right of the axis
 - b. 10 units to the right of the axis
 - c. $4\frac{1}{2}$ units to the left of the axis
 - d. 6 units to the left of the axis.
 - e. 2.5 units to the right of the axis.
 - f. 5.2 units to the left of the axis.
20. Which statement is true about the graph of the equation $y = 6$?
- (1) It is parallel to the y -axis
 - (2) It is parallel to the x -axis.
 - (3) It is *not* parallel to either axis.
 - (4) It goes through the origin.
21. Which statement is *not* true about the graph of the equation $x = 5$?
- (1) It goes through $(5, 0)$.
 - (2) It is a vertical line.
 - (3) It is a function.
 - (4) It is parallel to the y -axis.
22. Which statement is true about the graph of the equation $y = x$?
- (1) It is parallel to the y -axis.
 - (2) It is parallel to the x -axis.
 - (3) It goes through $(2, -2)$.
 - (4) It goes through the origin.

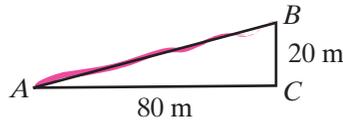
Applying Skills

23. The cost of admission to an amusement park on Family Night is \$25.00 for a family of any size.
- a. What is the cost of admission for the Gauger family of six persons?
 - b. Write an equation for the cost of admission, y , for a family of x persons.
24. The coordinates of the vertices of rectangle $ABCD$ are $A(-2, -1)$, $B(4, -1)$, $C(4, 5)$, and $D(-2, 5)$.
- a. Draw $ABCD$ on graph paper.
 - b. Write the equations of the lines \overleftrightarrow{AB} , \overleftrightarrow{BC} , \overleftrightarrow{CD} , and \overleftrightarrow{DA} .
 - c. Draw a horizontal line and a vertical line that separate $ABCD$ into four congruent rectangles.
 - d. Write the equations of the lines drawn in part c.
25. During the years 2004, 2005, and 2006, there were 5,432 AnyClothes stores.
- a. Write a linear equation that gives the number of stores, y .
 - b. Predict the number of stores for the years 2007 and 2008.
26. During the years 1999 through 2006, SmallTown News' circulation was 74,000 each year.
- a. Write a linear equation that gives the circulation, y .
 - b. Predict the circulation of SmallTown News in the years 2007 and 2008.

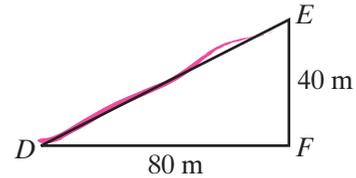
9-4 THE SLOPE OF A LINE

Meaning of the Slope of a Line

Easy Hill



Tough Hill



It is more difficult to hike up Tough Hill, shown above, than to hike up Easy Hill. Tough Hill rises 40 meters vertically over a horizontal distance of 80 meters, whereas Easy Hill rises only 20 meters vertically over the same horizontal distance of 80 meters. Therefore, Tough Hill is steeper than Easy Hill. To compare the steepness of roads \overline{AB} and \overline{DE} , which lead up the two hills, we compare their **slopes**.

The slope of road \overline{AB} is the ratio of the change in vertical distance, CB , to the change in horizontal distance, AC :

$$\text{slope of road } \overline{AB} = \frac{\text{change in vertical distance, } CB}{\text{change in horizontal distance, } AC} = \frac{20 \text{ m}}{80 \text{ m}} = \frac{1}{4}$$

Also:

$$\text{slope of road } \overline{DE} = \frac{\text{change in vertical distance, } FE}{\text{change in horizontal distance, } DF} = \frac{40 \text{ m}}{80 \text{ m}} = \frac{1}{2}$$

Note that the steeper hill has the larger slope.

Finding the Slope of a Line

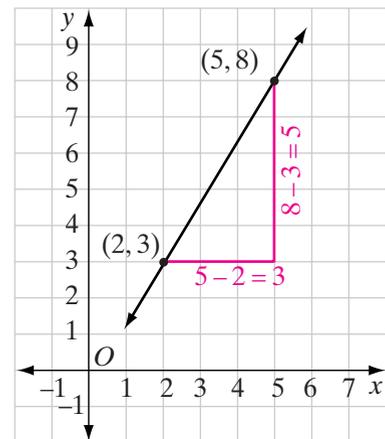
To find the slope of the line determined by the two points $(2, 3)$ and $(5, 8)$, as shown at the right, we write the ratio of the difference in y -values to the difference in x -values as follows:

$$\text{slope} = \frac{\text{difference in } y\text{-values}}{\text{difference in } x\text{-values}} = \frac{8-3}{5-2} = \frac{5}{3}$$

Suppose we change the order of the points $(2, 3)$ and $(5, 8)$ in performing the computation. We then have:

$$\text{slope} = \frac{\text{difference in } y\text{-values}}{\text{difference in } x\text{-values}} = \frac{3-8}{2-5} = \frac{-5}{-3} = \frac{5}{3}$$

The result of both computations is the same.



When we compute the slope of a line that is determined by two points, it does not matter which point is considered as the first point and which the second.

Also, when we find the slope of a line using two points on the line, it does not matter which two points on the line we use because all segments of a line have the same slope as the line.

Procedure

To find the slope of a line:

1. Select any two points on the line.
2. Find the vertical change, that is, the change in y -values by subtracting the y -coordinates in any order.
3. Find the horizontal change, that is, the change in x -values, by subtracting the x -coordinates in the same order as the y -coordinates.
4. Write the ratio of the vertical change to the horizontal change.

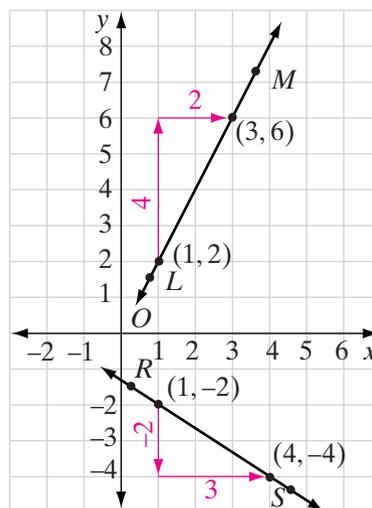
We can use this procedure to find the slopes of \overleftrightarrow{LM} and \overleftrightarrow{RS} , the lines shown in the graph at the right.

$$\begin{aligned}\text{slope of } \overleftrightarrow{LM} &= \frac{\text{vertical change}}{\text{horizontal change}} \\ &= \frac{6-2}{3-1} = \frac{4}{2} = \frac{2}{1} \text{ or } 2\end{aligned}$$

$$\begin{aligned}\text{slope of } \overleftrightarrow{RS} &= \frac{\text{vertical change}}{\text{horizontal change}} \\ &= \frac{-4 - (-2)}{4-1} = \frac{-2}{3} = -\frac{2}{3}\end{aligned}$$

Slope is a rate of change. It is a measure of the rate at which y changes compared to x . For \overleftrightarrow{LM} whose slope is 2 or $\frac{2}{1}$, y increases 2 units when x increases 1 unit. When the second element of a rate is 1, the rate is a **unit rate of change**. If x is the independent variable and y is the dependent variable, then the slope is the rate of change in the dependent variable compared to the independent variable.

For \overleftrightarrow{RS} whose slope is $-\frac{2}{3}$, y decreases 2 units when x increases 3 units. We could also write this rate as $-\frac{2}{1}$ to write the slope as a unit rate of change, that is, y decreases $\frac{2}{3}$ of a unit when x increases 1 unit.



In general:

- The slope, m , of a line that passes through any two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$, where $x_1 \neq x_2$, is the ratio of the change or difference of the y -values of these points to the change or difference of the corresponding x -values.

Thus:

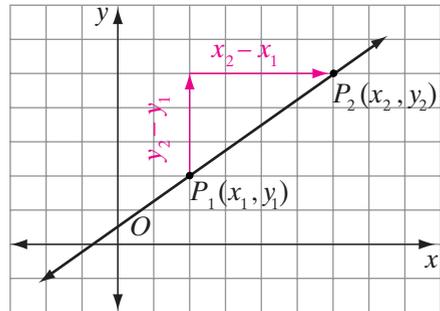
$$\text{slope of a line} = \frac{\text{difference in } y\text{-values}}{\text{difference in } x\text{-values}}$$

Therefore:

$$\text{slope of } \overleftrightarrow{P_1P_2} = m = \frac{y_2 - y_1}{x_2 - x_1}$$

The difference in x -values, $x_2 - x_1$, can be represented by Δx , read as “delta x .” Similarly, the difference in y -values, $y_2 - y_1$, can be represented by Δy , read as “delta y .” Therefore, we write:

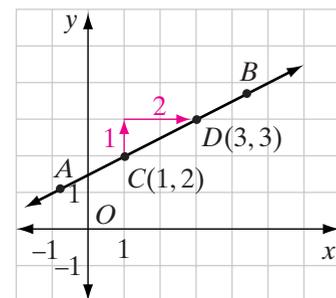
$$\text{slope of a line} = m = \frac{\Delta y}{\Delta x}$$



Positive Slopes

Examining \overleftrightarrow{AB} from left to right and observing the path of a point, from C to D for example, we see that the line is rising. As the x -values increase, the y -values also increase. Between point C and point D , the change in y is 1, and the change in x is 2. Since both Δy and Δx are positive, the slope of \overleftrightarrow{AB} must be positive. Thus:

$$\text{slope of } \overleftrightarrow{AB} = m = \frac{\Delta y}{\Delta x} = \frac{1}{2}$$

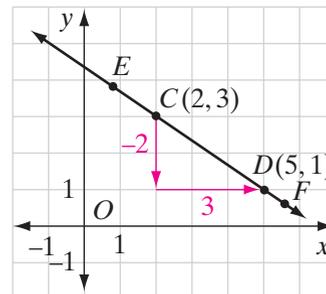


Principle 1. As a point moves from left to right along a line that is *rising*, y increases as x increases and the slope of the line is *positive*.

Negative Slopes

Now, examining \overleftrightarrow{EF} from left to right and observing the path of a point from C to D , we see that the line is falling. As the x -values increase, the y -values decrease. Between point C and point D , the change in y is -2 , and the change in x is 3 . Since Δy is negative and Δx is positive, the slope of \overleftrightarrow{EF} must be negative.

$$\text{slope of } \overleftrightarrow{EF} = m = \frac{\Delta y}{\Delta x} = \frac{-2}{3} = -\frac{2}{3}$$

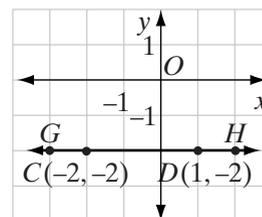


Principle 2. As a point moves from left to right along a line that is *falling*, y decreases as x increases and the slope of the line is *negative*.

Zero Slope

On the graph, \overleftrightarrow{GH} is parallel to the x -axis. We consider a point moving along \overleftrightarrow{GH} from left to right, for example from C to D . As the x -values increase, the y -values are unchanged. Between point C and point D , the change in y is 0 and the change in x is 3 . Since Δy is 0 and Δx is 3 , the slope of \overleftrightarrow{GH} must be 0 . Thus:

$$\text{slope of } \overleftrightarrow{GH} = m = \frac{\Delta y}{\Delta x} = \frac{0}{3} = 0$$

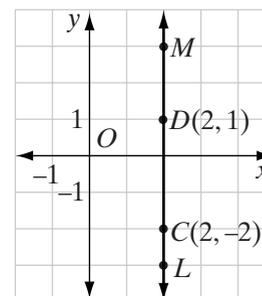


Principle 3. If a line is parallel to the x -axis, its slope is 0 .

Note: The slope of the x -axis is also 0 .

No Slope

On the graph, \overleftrightarrow{LM} is parallel to the y -axis. We consider a point moving upward along \overleftrightarrow{LM} , for example from C to D . The x -values are unchanged, but the y -values increase. Between point C and point D , the change in y is 3 , and the change in x is 0 . Since the slope of \overleftrightarrow{LM} is the change in y divided by the change in x and a number cannot be divided by 0 , \overleftrightarrow{LM} has no defined slope.



$$\text{slope of } \overleftrightarrow{MC} = m = \frac{\Delta y}{\Delta x} = \frac{1 - (-2)}{0} = \text{undefined}$$

Principle 4. If a line is parallel to the y-axis, it has no defined slope.

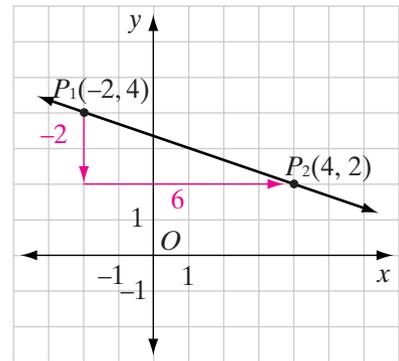
Note: The y-axis itself has no defined slope.

EXAMPLE 1

Find the slope of the line that is determined by points $(-2, 4)$ and $(4, 2)$.

Solution Plot points $(-2, 4)$ and $(4, 2)$. Let point $(-2, 4)$ be $P_1(x_1, y_1)$, and let point $(4, 2)$ be $P_2(x_2, y_2)$. Then, $x_1 = -2$, $y_1 = 4$, $x_2 = 4$, and $y_2 = 2$.

$$\begin{aligned} \text{slope of } \overleftrightarrow{P_1P_2} &= \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{2 - 4}{4 - (-2)} \\ &= \frac{2 - 4}{4 + 2} \\ &= \frac{-2}{6} \\ &= -\frac{1}{3} \quad \text{Answer} \end{aligned}$$

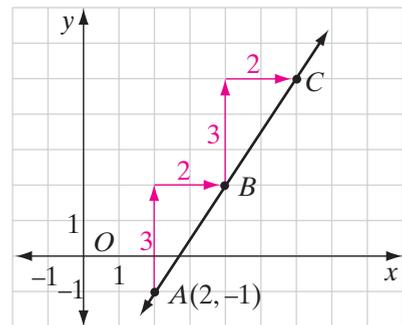


EXAMPLE 2

Through point $(2, -1)$, draw the line whose slope is $\frac{3}{2}$.

Solution *How to Proceed*

- (1) Graph point $A(2, -1)$:
- (2) Note that since slope $= \frac{\Delta y}{\Delta x} = \frac{3}{2}$, when y changes by 3, then x changes by 2. Start at point $A(2, -1)$ and move 3 units upward and 2 units to the right to locate point B :
- (3) Start at B and repeat these movements to locate point C :
- (4) Draw a line that passes through points A , B , and C :



KEEP IN MIND A fundamental property of a straight line is that its slope is constant. Therefore, any two points on a line may be used to compute the slope of the line.

Applications of Slope

In real-world contexts, the slope is either a ratio or a rate. When both the x and y variables have the same unit of measure, slope represents a *ratio* since it will have no unit of measure. However, when the x and y variables have different units of measure, slope represents a *rate* since it will have a unit of measure. In both cases, the slope represents a *constant* ratio or rate of change.

EXAMPLE 3

The following are slopes of lines representing the daily sales, y , over time, x , for various sales representatives during the course of a year:

- (1) $m = 20$ (2) $m = 45$
 (3) $m = 39$ (4) $m = -7$

- Interpret the meaning of the slopes if sales are given in thousands of dollars and time is given in months.
- Which sales representative had the greatest monthly increase in sales?
- Is it possible to determine which sales representative had the greatest total sales at the end of the year?

Solution a. The slopes represent the increase in sales *per month*:

- (1) \$20,000 increase each month (2) \$45,000 increase each month
 (3) \$39,000 increase each month (4) \$7,000 decrease each month

- At \$45,000, (2) has the greatest monthly increase in sales.
- The slopes give the *increase* in sales each month, not the total monthly sales. With the given information, it is not possible to determine which sales representative had the greatest total sales at the end of the year. ■

EXERCISES

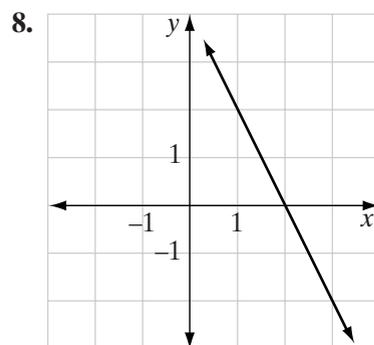
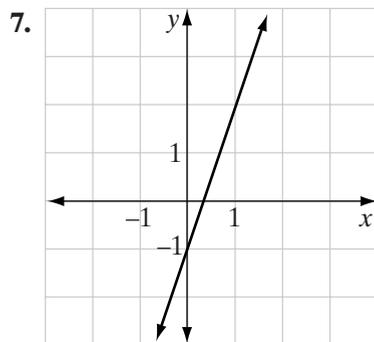
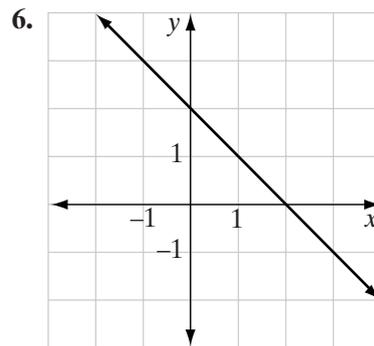
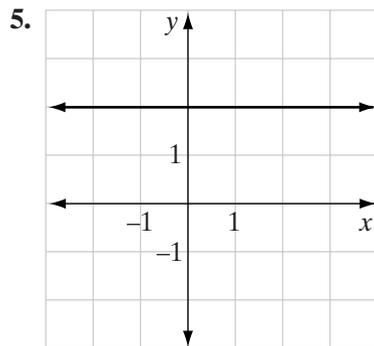
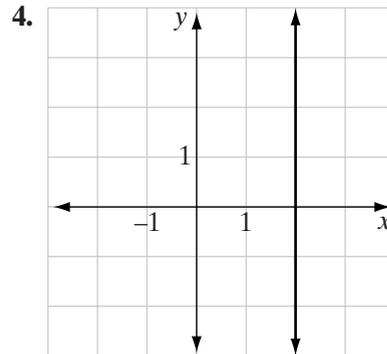
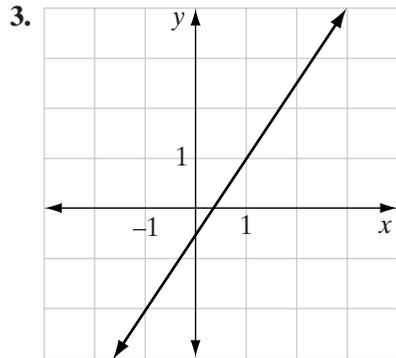
Writing About Mathematics

- Regina said that in Example 1, the answer would be the same if the formula for slope had been written as $\frac{y_1 - y_2}{x_1 - x_2}$. Do you agree with Regina? Explain why or why not. Use $\frac{y_1 - y_2}{x_1 - x_2}$ to find the answer to Example 1 to justify your response.

2. Explain why any two points that have the same x -coordinate lie on a line that has no slope.

Developing Skills

- In 3–8: **a.** Tell whether each line has a positive slope, a negative slope, a slope of zero, or no slope.
b. Find the slope of each line that has a slope.



In 9–17, in each case: **a.** Plot both points, and draw the line that they determine.

b. Find the slope of this line.

9. $(0, 0)$ and $(4, 4)$

10. $(0, 0)$ and $(4, 8)$

11. $(0, 0)$ and $(3, -6)$

12. $(1, 5)$ and $(3, 9)$

13. $(7, 3)$ and $(1, -1)$

14. $(-2, 4)$ and $(0, 2)$

15. $(5, -2)$ and $(7, -8)$

16. $(4, 2)$ and $(8, 2)$

17. $(-1, 3)$ and $(2, 3)$

In 18–29, in each case, draw a line with the given slope, m , through the given point.

18. $(0, 0)$; $m = 2$

19. $(1, 3)$; $m = 3$

20. $(2, -5)$; $m = 4$

21. $(-4, 5)$; $m = \frac{2}{3}$

22. $(3, 1)$; $m = 0$

23. $(-3, -4)$; $m = -2$

24. $(1, -5)$; $m = -1$

25. $(2, 4)$; $m = -\frac{3}{2}$

26. $(-2, 3)$; $m = -\frac{1}{3}$

27. $(-1, 0)$; $m = -\frac{5}{4}$

28. $(0, 2)$; $m = -\frac{2}{3}$

29. $(-2, 0)$; $m = \frac{1}{2}$

Applying Skills

30. Points $A(2, 4)$, $B(8, 4)$, and $C(5, 1)$ are the vertices of $\triangle ABC$. Find the slope of each side of $\triangle ABC$.

31. Points $A(3, -2)$, $B(9, -2)$, $C(7, 4)$, and $D(1, 4)$ are the vertices of a quadrilateral.

a. Graph the points and draw quadrilateral $ABCD$.

b. What type of quadrilateral does $ABCD$ appear to be?

c. Compute the slope of \overrightarrow{BC} and the slope of \overrightarrow{AD} .

d. What is true of the slope of \overrightarrow{BC} and the slope of \overrightarrow{AD} ?

e. If two segments such as \overline{AD} and \overline{BC} , or two lines such as \overleftrightarrow{AD} and \overleftrightarrow{BC} , are parallel, what appears to be true of their slopes?

f. Since \overline{AB} and \overline{CD} are parallel, what might be true of their slopes?

g. Compute the slope of \overline{AB} and the slope of \overline{CD} .

h. Is the slope of \overline{AB} equal to the slope of \overline{CD} ?

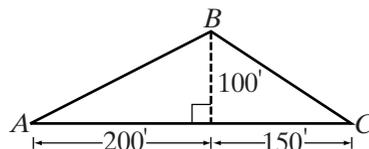
32. A path over a hill rises 100 feet vertically in a horizontal distance of 200 feet and then descends 100 feet vertically in a horizontal distance of 150 feet.

a. Find the slope of the path up the hill when walking from point A to point B .

b. Find the slope of the path down the hill when walking from point B to point C .

c. What is the unit rate of change from point A to point B ?

d. What is the unit rate of change from point B to point C ?



33. The amount that a certain internet phone company charges for international phone calls is represented by the equation $C = \$0.02t + \1.00 , where C represents the total cost of the call and t represents time in minutes. Explain the meaning of the slope in terms of the information provided.
34. The monthly utility bill for a certain school can be represented by the equation $C = \$0.075g + \100 , where C is the monthly bill and g is the amount of gas used by the hundred cubic feet (CCF). Explain the meaning of the slope in terms of the information provided.
35. A road that has an 8% grade rises 8 feet vertically for every 100 feet horizontally.
- Find the slope of the road.
 - Explain the meaning of the slope in terms of the information provided.
 - If you travel a horizontal distance of 200 feet on this road, what will be the amount of vertical change in your position?

9-5 THE SLOPES OF PARALLEL AND PERPENDICULAR LINES

Parallel Lines

In the diagram, lines ℓ_1 and ℓ_2 are parallel. Right triangle 1 has been drawn with one leg coinciding with the x -axis and with its hypotenuse coinciding with ℓ_1 . Similarly, right triangle 2 has been drawn with one leg coinciding with the x -axis and with its hypotenuse coinciding with ℓ_2 .

Since triangles 1 and 2 are right triangles,

$$\tan \angle a = \frac{\text{opp}_a}{\text{adj}_a} \quad \tan \angle b = \frac{\text{opp}_b}{\text{adj}_b}$$

The x -axis is a transversal that intersects parallel lines ℓ_1 and ℓ_2 . The corresponding angles, $\angle a$ and $\angle b$, are congruent. Therefore,

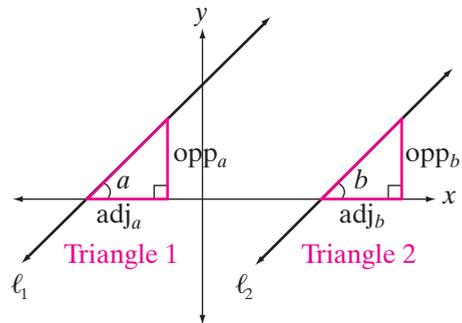
$$\frac{\text{opp}_a}{\text{adj}_a} = \frac{\text{opp}_b}{\text{adj}_b}$$

Since $\frac{\text{opp}_a}{\text{adj}_a}$ is the slope of ℓ_1 and $\frac{\text{opp}_b}{\text{adj}_b}$ is the slope of ℓ_2 , we have shown that two parallel lines have the same slope.

► If the slope of ℓ_1 is m_1 , the slope of ℓ_2 is m_2 , and $\ell_1 \parallel \ell_2$, then $m_1 = m_2$.

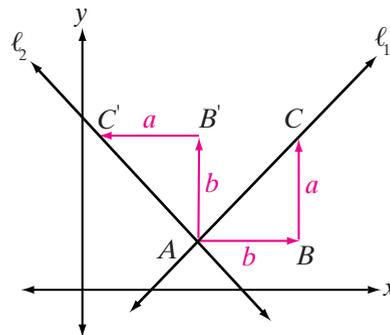
The following statement is also true:

► If the slope of ℓ_1 is m_1 , the slope of ℓ_2 is m_2 , and $m_1 = m_2$, then $\ell_1 \parallel \ell_2$.



Perpendicular Lines

In the diagram, lines ℓ_1 and ℓ_2 are perpendicular. Right triangle ABC has been drawn with one leg parallel to the x -axis, the other leg parallel to the y -axis, and the hypotenuse coinciding with ℓ_1 . The slope of ℓ_1 is $\frac{a}{b}$ since y increases by a units as x increases by b units. Triangle ABC is then rotated 90° counter-clockwise about A so that the hypotenuse of $AB'C'$ coincides with ℓ_2 .



Notice that since each leg has been rotated by 90° , the two legs are still parallel to the axes. However, they are now parallel to different axes. Also, in the rotated triangle, the change in y is an increase of b units and the change in x is a decrease of a units, that is, $-a$. Therefore, the slope of ℓ_2 is $\frac{b}{-a}$ or $-\frac{b}{a}$. In other words, the slope of ℓ_2 is the negative reciprocal of the slope of ℓ_1 . We have thus shown that perpendicular lines have slopes that are negative reciprocals of each other.

In general:

- If the slope of ℓ_1 is m_1 , the slope of ℓ_2 is m_2 , and $\ell_1 \perp \ell_2$, then $m_1 = -\frac{1}{m_2}$ or $m_1 \cdot m_2 = -1$.

The following statement is also true:

- If the slope of ℓ_1 is m_1 , the slope of ℓ_2 is m_2 , and $m_1 = -\frac{1}{m_2}$ or $m_1 \cdot m_2 = -1$, then $\ell_1 \perp \ell_2$.

EXAMPLE I

The equation of \overleftrightarrow{AB} is $y = -2x + 1$.

- Find the coordinates of any two points on \overleftrightarrow{AB} .
- What is the slope of \overleftrightarrow{AB} ?
- What is the slope of a line that is parallel to \overleftrightarrow{AB} ?
- What is the slope of a line that is perpendicular to \overleftrightarrow{AB} ?

Solution a. If $x = 0$, $y = -2(0) + 1 = 1$. One point is $(0, 1)$.

If $x = 1$, $y = -2(1) + 1 = -1$. Another point is $(1, -1)$.

- b. Slope of $\overleftrightarrow{AB} = \frac{1 - (-1)}{0 - 1} = \frac{2}{-1} = -2$.

- c. The slope of a line parallel to \overleftrightarrow{AB} is equal to the slope of \overleftrightarrow{AB} , -2 .
 d. The slope of a line perpendicular to \overleftrightarrow{AB} is $-\frac{1}{-2} = \frac{1}{2}$.

Answers a. $(0, 1)$ and $(1, -1)$ but many other answers are possible.

b. -2 c. -2 d. $\frac{1}{2}$



EXERCISES

Writing About Mathematics

- a. What is the slope of a line that is perpendicular to a line whose slope is 0? Explain your answer.

b. What is the slope of a line that is perpendicular to a line that has no slope? Explain your answer.
- What is the unit rate of change of y with respect to x if the slope of the graph of the equation for y in terms of x is $\frac{1}{4}$?

Developing Skills

In 3–11: **a.** Write the coordinates of two points on each line whose equation is given. **b.** Use the coordinates of the points found in part **a** to find the slope of the line. **c.** What is the slope of a line that is parallel to each line whose equation is given? **d.** What is the slope of a line that is perpendicular to the line whose equation is given?

3. $y = 2x + 6$

4. $y = x - 2$

5. $y = -3x + 7$

6. $3y = x$

7. $x - y = 4$

8. $2x - 3y = 6$

9. $x = 2y - 1$

10. $x = 4$

11. $y = -5$

Applying Skills

- A taxi driver charges \$3.00 plus \$0.75 per mile.
 - What is the cost of a trip of 4 miles?
 - What is the cost of a trip of 8 miles?
 - Use the information from parts **a** and **b** to draw the graph of a linear function described by the taxi fares.
 - Write an equation for y , the cost of a taxi ride, in terms of x , the length of the ride in miles.
 - What is the change in the cost of a taxi ride when the length of the ride changes by 4 miles?
 - What is the unit rate of change of the cost with respect to the length of the ride in dollars per mile?

13. Mrs. Boyko is waiting for her oven to heat to the required temperature. When she first turns the oven on, the temperature registers 100° . She knows that the temperature will increase by 10° every 5 seconds.
- What is the temperature of the oven after 10 seconds?
 - What is the temperature of the oven after 15 seconds?
 - Use the information from parts **a** and **b** to draw the graph of the linear function described by the oven temperatures.
 - Write an equation for y , the temperature of the oven, in terms of x , the number of seconds that it has been heating.
 - What is the unit rate in degrees per second at which the oven heats?

9-6 THE INTERCEPTS OF A LINE

The graph of any linear equation that is not parallel to an axis intersects both axes. The point at which the graph intersects the y -axis is a point whose x -coordinate is 0. The y -coordinate of this point is called the **y -intercept** of the equation. For example, for the equation $2x - y = 6$, let $x = 0$:

$$\begin{aligned} 2x - y &= 6 \\ 2(0) - y &= 6 \\ 0 - y &= 6 \\ -y &= 6 \\ y &= -6 \end{aligned}$$

The graph of $2x - y = 6$ intersects the y -axis at $(0, -6)$ and the y -intercept is -6 .

The point at which the graph intersects the x -axis is a point whose y -coordinate is 0. The x -coordinate of this point is called the **x -intercept** of the equation. For example, for the equation $2x - y = 6$, let $y = 0$:

$$\begin{aligned} 2x - y &= 6 \\ 2x - 0 &= 6 \\ 2x &= 6 \\ x &= 3 \end{aligned}$$

The graph of $2x - y = 6$ intersects the x -axis at $(3, 0)$ and the x -intercept is 3.

Divide each term of the equation $2x - y = 6$ by the constant term, 6

$$\begin{aligned} \frac{2x}{6} - \frac{y}{6} &= \frac{6}{6} \\ \frac{x}{3} - \frac{y}{6} &= 1 \end{aligned}$$

Note that x is divided by the x -intercept, 3, and y is divided by the y -intercept, -6 .

In general, if a linear equation is written in the form $\frac{x}{a} + \frac{y}{b} = 1$, the x -intercept is a and the y -intercept is b .

$$\frac{x}{a} + \frac{y}{b} = 1$$

x-intercept
y-intercept

An equation of the form $\frac{x}{a} + \frac{y}{b} = 1$ is called **the intercept form of a linear equation**. Two points on the graph of $\frac{x}{a} + \frac{y}{b} = 1$ are $(a, 0)$ and $(0, b)$. We can use these two points to find the slope of the line.

$$\text{slope} = \frac{0 - b}{a - 0} = \frac{-b}{a} = -\frac{b}{a}$$

Slope and y-Intercept

Consider the equation $2x - y = 6$ from another point of view. Solve the equation for y :

$$\begin{aligned} 2x - y &= 6 \\ -y &= -2x + 6 \\ y &= 2x - 6 \end{aligned}$$

There are two numbers in the equation that has been solved for y : the coefficient of x , 2, and the constant term, -6 . Each of these numbers gives us important information about the graph.

The y -intercept of a graph is the y -coordinate of the point at which the graph intersects the y -axis. For every point on the y -axis, $x = 0$. When $x = 0$,

$$\begin{aligned} y &= 2(0) - 6 \\ &= 0 - 6 \\ &= -6 \end{aligned}$$

Therefore, -6 is the y -intercept of the graph of the equation $y = 2x - 6$ and $(0, -6)$ is a point on its graph. The y -intercept is the constant term of the equation when the equation is solved for y .

Another point on the graph of $y = 2x - 6$ is $(1, -4)$. Use the two points, $(0, -6)$ and $(1, -4)$, to find the slope of the line.

$$\begin{aligned} \text{slope of } y = 2x - 6 &= \frac{-6 - (-4)}{0 - 1} \\ &= \frac{-2}{-1} \\ &= 2 \end{aligned}$$

The slope is the coefficient of x when the equation is solved for y .

For any equation in the form $y = mx + b$:

1. The coefficient of x is the slope of the line.
2. The constant term is the y -intercept.

The equation we have been studying can be compared to a general equation of the same type:

$$\begin{array}{ccc}
 y = 2x - 3 & & y = mx + b \\
 \swarrow \quad \nwarrow & & \swarrow \quad \nwarrow \\
 \text{slope} \quad \quad \text{y-intercept} & & \text{slope} \quad \quad \text{y-intercept}
 \end{array}$$

The following statement is true for the general equation:

► **If the equation of a line is written in the form $y = mx + b$, then the slope of the line is m and the y-intercept is b .**

An equation of the form $y = mx + b$ is called **the slope-intercept form of a linear equation**.

EXAMPLE 1

Find the intercepts of the graph of the equation $5x + 2y = 6$.

Solution To find the x intercept, let $y = 0$:

$$\begin{aligned}
 5x + 2y &= 6 \\
 5x + 2(0) &= 6 \\
 5x + 0 &= 6 \\
 5x &= 6 \\
 x &= \frac{6}{5}
 \end{aligned}$$

To find the y intercept, let $x = 0$:

$$\begin{aligned}
 5x + 2y &= 6 \\
 5(0) + 2y &= 6 \\
 0 + 2y &= 6 \\
 2y &= 6 \\
 y &= 3
 \end{aligned}$$

Answer The x intercept is $\frac{6}{5}$ and the y intercept is 3. ■

EXAMPLE 2

Find the slope and the y -intercept of the line that is the graph of the equation $4x + 2y = 10$.

Solution

How to Proceed

(1) Solve the given equation for y to obtain an equivalent equation of the form $y = mx + b$:

$$\begin{aligned}
 4x + 2y &= 10 \\
 2y &= -4x + 10 \\
 y &= -2x + 5
 \end{aligned}$$

(2) The coefficient of x is the slope:

$$\text{slope} = -2$$

(3) The constant term is the y -intercept:

$$\text{y-intercept} = 5$$

Answer slope = -2 , y -intercept = 5 ■

EXERCISES

Writing About Mathematics

1. Amanda said that the y -intercept of the line whose equation is $2y + 3x = 8$ is 8 because the y -intercept is the constant term. Do you agree with Amanda? Explain why or why not.
2. Roberto said that the line whose equation is $y = x$ has no slope because x has no coefficient and no y -intercept because there is no constant term. Do you agree with Roberto? Explain why or why not.

Developing Skills

In 3–18, in each case, find the slope, y -intercept, and x -intercept of the line that is the graph of the equation.

- | | | | |
|-------------------|--------------------|-------------------------------------|---|
| 3. $y = 3x + 1$ | 4. $y = 3 - x$ | 5. $y = 2x$ | 6. $y = x$ |
| 7. $x = -3$ | 8. $y = -2$ | 9. $y = -\frac{2}{3}x + 4$ | 10. $y - 3x = 7$ |
| 11. $2x + y = 5$ | 12. $3y = 6x + 9$ | 13. $2y = 5x - 4$ | 14. $\frac{1}{2}x + \frac{3}{4} = \frac{1}{3}y$ |
| 15. $4x - 3y = 0$ | 16. $2x = 5y - 10$ | 17. $\frac{x}{2} + \frac{y}{3} = 1$ | 18. $\frac{4x}{5} - 2y = 1$ |
19. What do the graphs of the equations $y = 4x$, $y = 4x + 2$, and $y = 4x - 2$ all have in common?
 20. What do the lines that are the graphs of the equations $y = 2x + 1$, $y = 3x + 1$, and $y = -4x + 1$ all have in common?
 21. If two lines are parallel, how are their slopes related?
 22. What is true of two lines whose slopes are equal?

In 23–26, state in each case whether the lines are parallel, perpendicular, or neither.

- | | |
|---------------------------------|----------------------------------|
| 23. $y = 3x + 2$, $y = 3x - 5$ | 24. $y = -2x - 6$, $y = 2x + 6$ |
| 25. $y = 4x - 8$, $4y + x = 3$ | 26. $y = 2x$, $x = -2y$ |
27. Which of the following statements is true of the graph of $y = -3x$?
 - (1) It is parallel to the x -axis.
 - (2) It is parallel to the y -axis.
 - (3) Its slope is -3 .
 - (4) It does not have a y -intercept.
 28. Which of the following statements is true of the graph of $y = 8$?
 - (1) It is parallel to the x -axis.
 - (2) It is parallel to the y -axis.
 - (3) It has no slope.
 - (4) It goes through the origin.

9-7 GRAPHING LINEAR FUNCTIONS USING THEIR SLOPES

The slope and any one point can be used to draw the graph of a linear function. One convenient point is the point of intersection with the y -axis, that is, the point whose x -coordinate is 0 and whose y -coordinate is the y -intercept.

EXAMPLE I

Draw the graph of $2x + 3y = 9$ using the slope and the y -intercept.

Solution *How to Proceed*

- (1) Transform the equation into the form $y = mx + b$:
- (2) Find the slope of the line (the coefficient of x):
- (3) Find the y -intercept of the line (the constant):
- (4) On the y -axis, graph point A , whose y -coordinate is the y -intercept:
- (5) Use the slope to find two more points on the line. Since slope $= \frac{\Delta y}{\Delta x} = \frac{-2}{3}$, when y changes by -2 , x changes by 3. Therefore, start at point A and move 2 units down and 3 units to the right to locate point B . Then start at point B and repeat the procedure to locate point C :
- (6) Draw the line that passes through the three points:

This line is the graph of $2x + 3y = 9$.

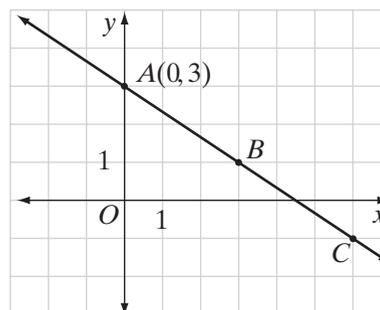
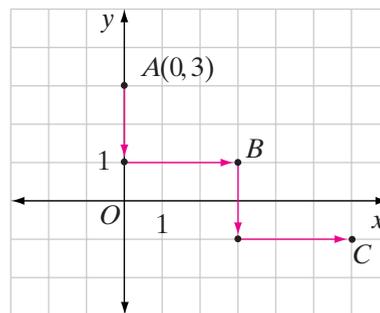
Check To check a graph, select two or more points on the line drawn and substitute their coordinates in the original equation. For example, check this graph using points $(4.5, 0)$, $(3, 1)$, and $(6, -1)$.

$$\begin{aligned} 2x + 3y &= 9 \\ 3y &= -2x + 9 \end{aligned}$$

$$y = \frac{-2}{3}x + 3$$

$$\text{slope} = \frac{-2}{3}$$

$$y\text{-intercept} = 3$$



$$\begin{array}{rcl}
 2x + 3y = 9 & 2x + 3y = 9 & 2x + 3y = 9 \\
 2(4.5) + 3(0) \stackrel{?}{=} 9 & 2(3) + 3(1) \stackrel{?}{=} 9 & 2(6) + 3(-1) \stackrel{?}{=} 9 \\
 9 + 0 \stackrel{?}{=} 9 & 6 + 3 \stackrel{?}{=} 9 & 12 - 3 \stackrel{?}{=} 9 \\
 9 = 9 \checkmark & 9 = 9 \checkmark & 9 = 9 \checkmark
 \end{array}$$

In this example, we used the slope and the y -intercept to draw the graph. We could use any point on the graph of the equation as a starting point.

EXAMPLE 2

Draw the graph of $3x - 2y = 9$ using the slope and any point.

Solution

How to Proceed

- (1) Solve the equation for y . The slope is $\frac{3}{2}$ and the y -intercept is $-\frac{9}{2}$:

$$\begin{aligned}
 3x - 2y &= 9 \\
 -2y &= -3x + 9 \\
 y &= \frac{-3x}{-2} + \frac{9}{-2} \\
 y &= \frac{3}{2}x - \frac{9}{2} \\
 y &= \frac{3}{2}(1) - \frac{9}{2} \\
 y &= \frac{3}{2} - \frac{9}{2} \\
 y &= -\frac{6}{2} \\
 y &= -3
 \end{aligned}$$

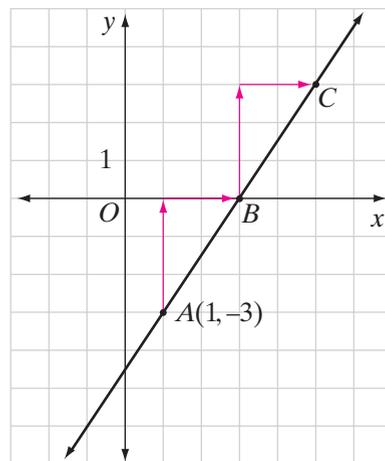
- (2) Since $(0, -\frac{9}{2})$ is not a convenient point to graph, choose another point.

Let $x = 1$. A convenient point on the graph is $A(1, -3)$:

- (3) Graph point A whose coordinates are $(1, -3)$:

- (4) Use the slope to find two more points on the line. Since slope $= \frac{\Delta y}{\Delta x} = \frac{3}{2}$, when y changes by 3, x changes by 2. Therefore, start at point A and move 3 units up and 2 units to the right to locate point B . Then start at point B and repeat the procedure to locate point C :

- (5) Draw the line that passes through the three points. This is the graph of $3x - 2y = 9$:

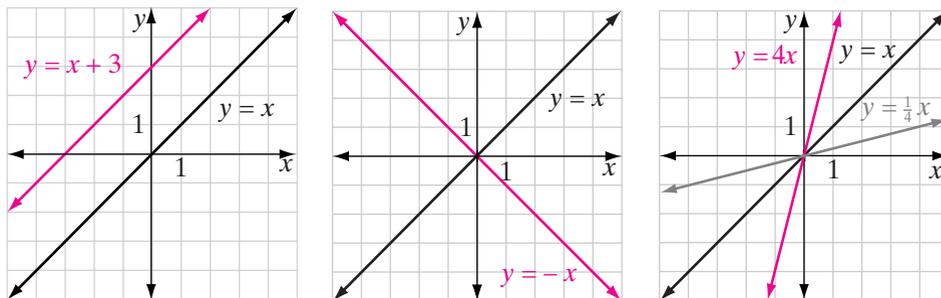


Note that the graph intersects the x -axis at $B(3, 0)$. The x -intercept is 3.

Translating, Reflecting, and Scaling Graphs of Linear Functions

An alternative way of looking at the effects of changing the values of the y -intercept and slope is that of translating, reflecting, or scaling the graph of the linear function $y = x$.

For instance, the graph of $y = x + 3$ can be thought of as the graph of $y = x$ shifted 3 units up. Similarly, the graph of $y = -x$ can be thought of the graph of $y = x$ reflected in the x -axis. On the other hand, the graph of $y = 4x$ can be thought of as the graph of $y = x$ stretched vertically by a factor of 4, while the graph of $y = \frac{1}{4}x$ is the graph of $y = x$ compressed vertically by a factor of $\frac{1}{4}$.



Translation Rules for Linear Functions

If c is positive:

- ▶ The graph of $y = x + c$ is the graph of $y = x$ shifted c units up.
- ▶ The graph of $y = x - c$ is the graph of $y = x$ shifted c units down.

Reflection Rule for Linear Functions

- ▶ The graph of $y = -x$ is the graph of $y = x$ reflected in the x -axis.

Scaling Rules for Linear Functions

- ▶ When $c > 1$, the graph of $y = cx$ is the graph of $y = x$ stretched vertically by a factor of c .
- ▶ When $0 < c < 1$, the graph of $y = cx$ is the graph of $y = x$ compressed vertically by a factor of c .

EXAMPLE 3

In **a–d**, write an equation for the resulting function if the graph of $y = x$ is:

- a.** shifted 6 units down
- b.** reflected in the x -axis and shifted 3 units up
- c.** compressed vertically by a factor of $\frac{1}{10}$ and shifted 1 unit up
- d.** stretched vertically by a factor of 4 and shifted 1 unit down

Solution **a.** $y = x - 6$ *Answer*

b. First, reflect in the x -axis:

$$y = -x$$

Then, translate the resulting function 3 units up:

$$y = -x + 3 \text{ *Answer*}$$

c. First, compress vertically by a factor of $\frac{1}{10}$:

$$y = \frac{1}{10}x$$

Then, translate the resulting function 1 unit up:

$$y = \frac{1}{10}x + 1 \text{ *Answer*}$$

d. First, stretch vertically by a factor of 4:

$$y = 4x$$

Then, translate the resulting function 1 unit down:

$$y = 4x - 1 \text{ *Answer*}$$

EXERCISES**Writing About Mathematics**

- Explain why $(0, b)$ is always a point on the graph of $y = mx + b$.
- Gunther said that in the first example in this section, since the slope of the line, $-\frac{2}{3}$, could have been written as $\frac{2}{-3}$, points on the line could have been located by moving up 2 units and to the left 3 units from the point $(0, 3)$. Do you agree with Gunther? Explain why or why not.
- Hypatia said that, for linear functions, vertical translations have the same effect as *horizontal* translations and that reflecting across the x -axis has the same effect as reflecting across the y -axis. Do you agree with Hypatia? Explain why or why not.

Developing Skills

In 4–15, graph each equation using the slope and the y -intercept.

4. $y = 2x + 3$

5. $y = 2x$

6. $y = -2x$

7. $y = -3x - 2$

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8. $y = \frac{2}{3}x + 2$

9. $y = \frac{1}{2}x - 1$

10. $y = -\frac{1}{3}x$

11. $y = -\frac{3}{4}x + 6$

12. $y - 2x = 8$

13. $3x + y = 4$

14. $\frac{x}{2} + \frac{y}{3} = 1$

15. $4x = 2y$

In 16–21, graph the equation using the slope and any point.

16. $3y = 4x + 9$

17. $5x - 2y = 3$

18. $3x + 4y = 7$

19. $2x = 3y - 1$

20. $3y - x = 5$

21. $2x - 3y - 6 = 0$

In 22–25, describe the translation, reflection, and/or scaling that must be applied to $y = x$ to obtain the graph of each given function.

22. $y = -x + 2$

23. $y = -\frac{1}{3}x - 2$

24. $y = 2x + 1.5$

25. $y = (x + 2) + 2$

Applying Skills

26. **a.** Draw the line through $(-2, -3)$ whose slope is 2.
b. What appears to be the y -intercept of this line?
c. Use the slope of the line and the answer to part **b** to write an equation of the line.
d. Do the coordinates of point $(-2, -3)$ satisfy the equation written in part **c**?
27. **a.** Draw the line through $(3, 5)$ whose slope is $\frac{2}{3}$.
b. What appears to be the y -intercept of this line?
c. Use the slope of the line and the answer to part **b** to write an equation of the line.
d. Do the coordinates of point $(3, 5)$ satisfy the equation written in part **c**?
28. **a.** Is $(1, 1)$ a point on the graph of $3x - 2y = 1$?
b. What is the slope of $3x - 2y = 1$?
c. Draw the graph of $3x - 2y = 1$ using point $(1, 1)$ and the slope of the line.
d. Why is it easier to use point $(1, 1)$ rather than the y -intercept to draw this graph?

9-8 GRAPHING DIRECT VARIATION

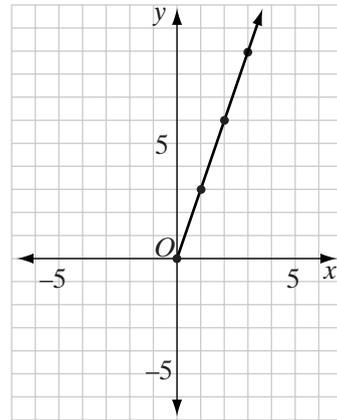
Recall that, when two variables represent quantities that are directly proportional or that vary directly, the ratio of the two variables is a constant. For example, when lemonade is made from a frozen concentrate, the amount of lemonade, y , that is obtained varies directly as the amount of concentrate, x , that is used. If we can make 3 cups of lemonade by using 1 cup of concentrate, the relationship can be expressed as

$$\frac{y}{x} = \frac{3}{1} \quad \text{or} \quad y = 3x$$

The *constant of variation* is 3.

The equation $y = 3x$ can be represented by a line in the coordinate plane, as shown in the graph below.

x	$3x$	y
0	$3(0)$	0
1	$3(1)$	3
2	$3(2)$	6
3	$3(3)$	9



Here, the replacement set for x and y is the set of positive numbers and 0. Thus, the graph includes only points in the first quadrant.

In our example, if 0 cups of frozen concentrate are used, 0 cups of lemonade can be made. Thus, the ordered pair $(0, 0)$ is a member of the solution set of $y = 3x$. Using any two points from the table above, for example, $(0, 0)$ and $(1, 3)$, we can write:

$$\frac{\Delta y}{\Delta x} = \frac{3 - 0}{1 - 0} = \frac{3}{1} = 3$$

Thus, we see that the slope of the line is also the constant of variation. An **equation of a direct variation** is a special case of an equation written in slope-intercept form; that is, $b = 0$ and $m =$ the constant of variation, k . A direct variation indicates that y is a multiple of x . Note that the graph of a direct variation is always a line through the origin.

The unit of measure for the lemonade and for the frozen concentrate is the same, namely, cups. Therefore, no unit of measure is associated with the ratio which, in this case, is $\frac{3}{1}$ or 3.

$$\frac{\text{lemonade}}{\text{concentrate}} = \frac{3 \text{ cups}}{1 \text{ cup}} = \frac{3}{1} \text{ or } \frac{\text{lemonade}}{\text{concentrate}} = \frac{6 \text{ cups}}{2 \text{ cups}} = \frac{3}{1} \text{ or } \frac{\text{lemonade}}{\text{concentrate}} = \frac{9 \text{ cups}}{3 \text{ cups}} = \frac{3}{1}$$

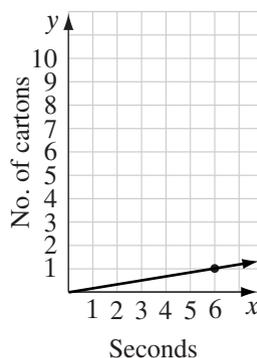
There are many applications of direct variation in business and science, and it is important to recognize how the choice of unit can affect the constant of variation. For example, if a machine is used to pack boxes of cereal in cartons, the rate at which the machine works can be expressed in cartons per minute or in cartons per second. If the machine fills a carton every 6 seconds, it will fill 10 cartons in 1 minute. The rate can be expressed as:

$$\frac{1 \text{ carton}}{6 \text{ seconds}} = \frac{1}{6} \text{ carton/second} \quad \text{or} \quad \frac{10 \text{ cartons}}{1 \text{ minute}} = 10 \text{ cartons/minute}$$

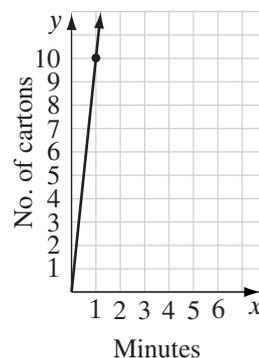
Here each rate has been written as a unit rate.

As shown at right, each of these rates can be represented by a graph, where the rate, or constant of variation, is the slope of the line.

The legend of the graph, that is, the units in which the rate is expressed, must be clearly stated if the graph is to be meaningful.



Cartons per second



Cartons per minute

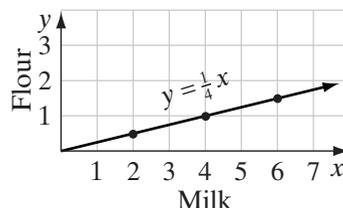
EXAMPLE 1

The amount of flour needed to make a white sauce varies directly as the amount of milk used. To make a white sauce, a chef used 2 cups of flour and 8 cups of milk. Write an equation and draw the graph of the relationship.

Solution Let x = the number of cups of milk,
and y = the number of cups of flour.
Then:

$$\begin{aligned}\frac{y}{x} &= \frac{2}{8} \\ 8y &= 2x \\ y &= \frac{2}{8}x \\ y &= \frac{1}{4}x\end{aligned}$$

x	$\frac{1}{4}x$	y
2	$\frac{1}{4}(2)$	$\frac{1}{2}$
4	$\frac{1}{4}(4)$	1
6	$\frac{1}{4}(6)$	$\frac{3}{2}$



EXERCISES

Writing About Mathematics

- A chef made the white sauce described in Example 1, measuring the flour and milk in ounces. Explain why the ratio of flour to milk should still be 1 : 4.
- When Eduardo made white sauce, he used 1 cup of flour and 1 quart of milk. Is the ratio of flour to milk in Eduardo's recipe the same as in the recipe used by the chef in the example? Explain why or why not.

Developing Skills

In 3–12, y varies directly as x . In each case: **a.** What is the constant of variation? **b.** Write an equation for y in terms of x . **c.** Using an appropriate scale, draw the graph of the equation written in part **b.** **d.** What is the slope of the line drawn in part **c.**

- The perimeter of a square (y) is 12 centimeters when the length of a side of the square (x) is 3 centimeters.

4. Jeanne can type 90 words (y) in 2 minutes (x).
5. A printer can type 160 characters (y) in 10 seconds (x).
6. A cake recipe uses 2 cups of flour (y) to $1\frac{1}{2}$ cups of sugar (x).
7. The length of a photograph (y) is 12 centimeters when the length of the negative from which it is developed (x) is 1.2 centimeters.
8. There are 20 slices (y) in 12 ounces of bread (x).
9. Three pounds of meat (y) will serve 15 people (x).
10. Twelve slices of cheese (y) weigh 8 ounces (x).
11. Willie averages 3 hits (y) for every 12 times at bat (x).
12. There are about 20 calories (y) in three crackers (x).

Applying Skills

13. If a car travels at a constant rate of speed, the distance that it travels varies directly as time. If a car travels 75 miles in 2.5 hours, it will travel 110 feet in 2.5 seconds.
 - a. Find the constant of variation in miles per hour.
 - b. Draw a graph that compares the distance that the car travels in miles to the number of hours traveled. Let the horizontal axis represent hours and the vertical axis represent distance.
 - c. Find the constant of variation in feet per second.
 - d. Draw a graph that compares the distance the car travels in feet to the number of seconds traveled.
14. In typing class, Russ completed a speed test in which he typed 420 characters in 2 minutes. His teacher told him to let 5 characters equal 1 word.
 - a. Find Russ's rate on the test in characters per second.
 - b. Draw a graph that compares the number of characters Russ typed to the number of minutes that he typed. (Let the horizontal axis represent minutes and the vertical axis represent characters.)
 - c. Find Russ's rate on the test in words per minute.
 - d. Draw a graph that compares the number of words Russ typed to the number of minutes that he typed. (Let the horizontal axis represent minutes and the vertical axis represent words.)

In 15–20, determine if the two variables are directly proportional. If so, write the equation of variation.

15. Perimeter of a square (P) and the length of a side (s).
16. Volume of a sphere (V) and radius (r).
17. The total amount tucked away in a piggy bank (s) and weekly savings (w), starting with an initial balance of \$50.

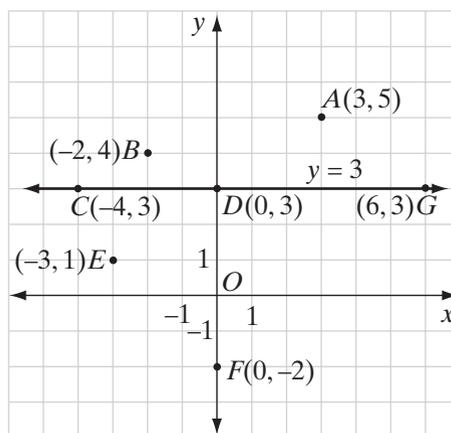
18. Simple interest on an investment (I) in one year and the amount invested (P), at a rate of 2.5%.
19. Length measured in centimeters (c) and length measured in inches (i).
20. Total distance traveled (d) in one hour and average speed (s).

9-9 GRAPHING FIRST-DEGREE INEQUALITIES IN TWO VARIABLES

When a line is graphed in the coordinate plane, the line is a **plane divider** because it separates the plane into two regions called **half-planes**. One of these regions is a half-plane on one side of the line; the other is a half-plane on the other side of the line.

Let us consider, for example, the horizontal line $y = 3$ as a plane divider. As shown in the graph below, the line $y = 3$ and the two half-planes that it forms determine three sets of points:

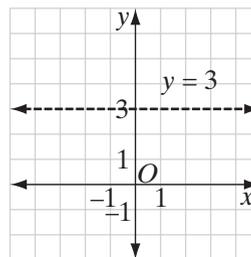
1. The half-plane above the line $y = 3$ is the set of all points whose y -coordinates are greater than 3, that is, $y > 3$. For example, at point A , $y = 5$; at point B , $y = 4$.
2. The line $y = 3$ is the set of all points whose y -coordinates are equal to 3. For example, $y = 3$ at each point $C(-4, 3)$, $D(0, 3)$, and $G(6, 3)$.
3. The half-plane below the line $y = 3$ is the set of all points whose y -coordinates are less than 3, that is, $y < 3$. For example, at point E , $y = 1$; at point F , $y = -2$.



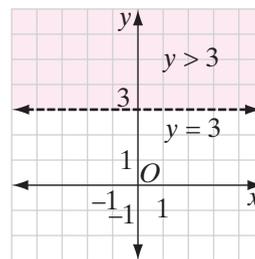
Together, the three sets of points form the entire plane.

To graph an inequality in the coordinate plane, we proceed as follows:

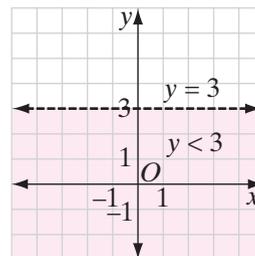
1. On the plane, represent the plane divider, for example, $y = 3$, by a *dashed line* to show that this divider does not belong to the graph of the half-plane.



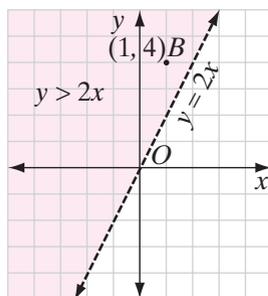
2. Shade the region of the half-plane whose points satisfy the inequality. To graph $y > 3$, shade the region above the plane divider.


 Graph of $y > 3$

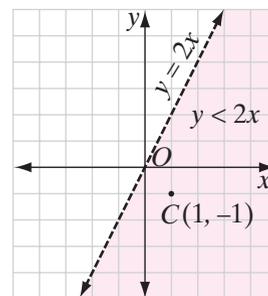
3. To graph $y < 3$, shade the region below the plane divider.


 Graph of $y < 3$

Let us consider another example, where the plane divider is not a horizontal line. To graph the inequality $y > 2x$ or the inequality $y < 2x$, we use a dashed line to indicate that the line $y = 2x$ is not a part of the graph. This dashed line acts as a boundary line for the half-plane being graphed.

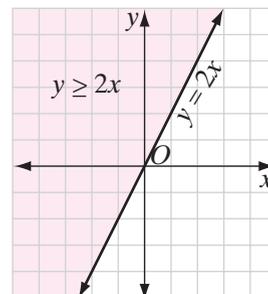

 Graph of $y > 2x$

The graph of $y > 2x$ is the shaded half-plane above the line $y = 2x$. It is the set of all points in which the y -coordinate is greater than twice the x -coordinate.


 Graph of $y < 2x$

The graph of $y < 2x$ is the shaded half-plane below the line $y = 2x$. It is the set of all points in which the y -coordinate is less than twice the x -coordinate.

An open sentence such as $y \geq 2x$ means $y > 2x$ or $y = 2x$. The graph of $y \geq 2x$ is the union of two disjoint sets. It includes all of the points in the solution set of the inequality $y > 2x$ and all of the points in the solution set of the equality $y = 2x$. To indicate that $y = 2x$ is part of the graph of $y \geq 2x$, we draw the graph of $y = 2x$ as a solid line. Then we shade the region above the line to include the points for which $y > 2x$.

Graph of $y \geq 2x$

► When the equation of a line is written in the form $y = mx + b$, the half-plane above the line is the graph of $y > mx + b$ and the half-plane below the line is the graph of $y < mx + b$.

To check whether the correct half-plane has been chosen as the graph of a linear inequality, we select any point in that half-plane. If the selected point satisfies the inequality, every point in that half-plane satisfies the inequality. On the other hand, if the point chosen does not satisfy the inequality, then the other half-plane is the graph of the inequality.

EXAMPLE I

Graph the inequality $y - 2x \geq 2$.

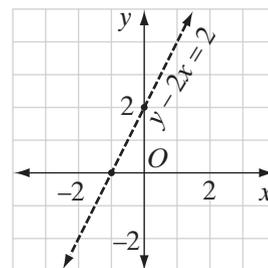
Solution

How to Proceed

- (1) Transform the inequality into one having y as the left member:
- (2) Graph the plane divider, $y = 2x + 2$, by using the y -intercept, 2, to locate the first point $(0, 2)$ on the y -axis. Then use the slope, 2 or $\frac{2}{1}$, to find other points by moving up 2 and to the right 1:

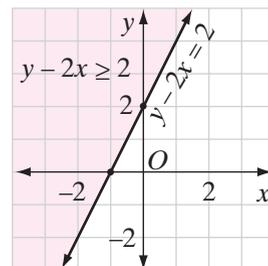
$$y - 2x \geq 2$$

$$y \geq 2x + 2$$



- (3) Shade the half-plane above the line:

This region and the line are the required graph. The half-plane is the graph of $y - 2x > 2$, and the line is the graph of $y - 2x = 2$. Note that the line is now drawn solid to show that it is part of the graph.



- (4) Check the solution. Choose any point in the half-plane selected as the solution to see whether it satisfies the original inequality, $y - 2x \geq 2$:

Select point $(0, 5)$ which is in the shaded region.

$$y - 2x \geq 2$$

$$5 - 2(0) \stackrel{?}{\geq} 2$$

$$5 \geq 2 \checkmark$$

The above graph is the graph of $y - 2x \geq 2$.

EXAMPLE 2

Graph each of the following inequalities in the coordinate plane.

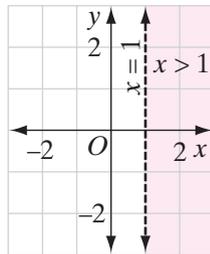
a. $x > 1$

b. $x \leq 1$

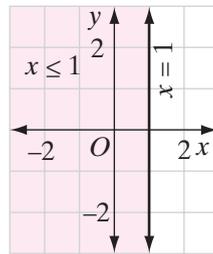
c. $y \geq 1$

d. $y < 1$

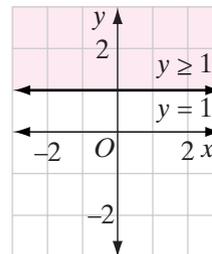
Answers



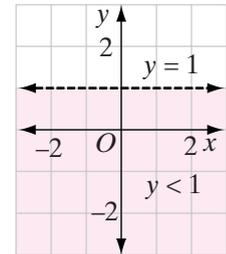
a. $x > 1$



b. $x \leq 1$



c. $y \geq 1$



d. $y < 1$

EXERCISES

Writing About Mathematics

- Brittany said that the graph of $2x - y > 5$ is the region above the line that is the graph of $2x - y = 5$. Do you agree with Brittany? Explain why or why not.
- Brian said that the union of the graph of $x > 2$ and the graph of $x < 2$ consists of every point in the coordinate plane. Do you agree with Brian? Explain why or why not.

Developing Skills

In 3–8, transform each sentence into one whose left member is y .

3. $y - 2x > 0$

4. $5x > 2y$

5. $y - x \geq 3$

6. $2x + y \leq 0$

7. $3x - y \geq 4$

8. $4y - 3x \leq 12$

In 9–23, graph each sentence in the coordinate plane.

- | | | |
|---------------------|--|---------------------------------------|
| 9. $x > 4$ | 10. $x \leq -2$ | 11. $y > 5$ |
| 12. $y \leq -3$ | 13. $y < x - 2$ | 14. $y \geq \frac{1}{2}x + 3$ |
| 15. $x + y \leq -3$ | 16. $x - y \leq -1$ | 17. $x - 2y \leq 4$ |
| 18. $2y - 6x > 0$ | 19. $2x - 3y \geq 6$ | 20. $\frac{1}{2}y > \frac{1}{3}x + 1$ |
| 21. $9 - x \geq 3y$ | 22. $\frac{x}{2} + \frac{y}{5} \leq 1$ | 23. $2x + 2y - 6 \leq 0$ |

In 24–26: **a.** Write each verbal sentence as an open sentence. **b.** Graph each open sentence in the coordinate plane.

24. The y -coordinate of a point is equal to or greater than 3 more than the x -coordinate.
25. The sum of the x -coordinate and the y -coordinate of a point is less than or equal to 5.
26. The y -coordinate of a point decreased by 3 times the x -coordinate is greater than or equal to 2.

Applying Skills

In 27–31: **a.** Write each verbal sentence as an open sentence. **b.** Graph each open sentence in the coordinate plane. **c.** Choose one pair of coordinates that could be reasonable values for x and y .

27. The length of Mrs. Gauger's garden (y) is greater than the width (x).
28. The cost of a shirt (y) is less than half the cost of a pair of shoes (x).
29. The distance to school (y) is at least 2 miles more than the distance to the library (x).
30. The height of the flagpole (y) is at most 4 feet more than the height of the oak tree (x) nearby.
31. At the water park, the cost of a hamburger (x) plus the cost of a can of soda (y) is greater than 5 dollars.

9-10 GRAPHS INVOLVING ABSOLUTE VALUE

To draw the graph of the equation $y = |x|$, we can choose values of x and then find the corresponding values of y .

Let us consider the possible choices for x and the resulting y -values:

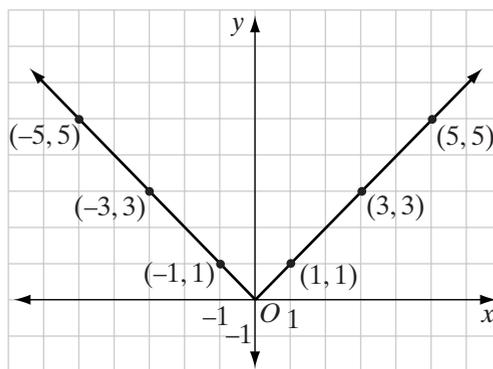
1. Choose $x = 0$. Since the absolute value of 0 is 0, y will be 0.
2. Choose x as any positive number. Since the absolute value of any positive number is that positive number, y will have the same value as x . For example, if $x = 5$, then $y = |5| = 5$.

3. Choose x as any negative number. Since the absolute value of any negative number is positive, y will be the opposite of x . For example, if $x = -3$, then $y = |-3| = 3$.

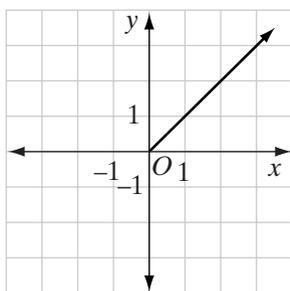
Thus, we conclude that x can be 0, positive, or negative, but y will be only 0 or positive.

Here is a table of values and the corresponding graph:

x	$ x $	y
-5	$ -5 $	5
-3	$ -3 $	3
-1	$ -1 $	1
0	$ 0 $	0
1	$ 1 $	1
3	$ 3 $	3
5	$ 5 $	5

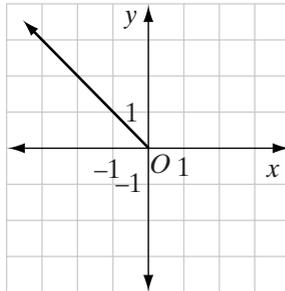


Notice that for positive values of x , the graph of $y = |x|$ is the same as the graph of $y = x$. For negative values of x , the graph of $y = |x|$ is the same as the graph of $y = -x$.



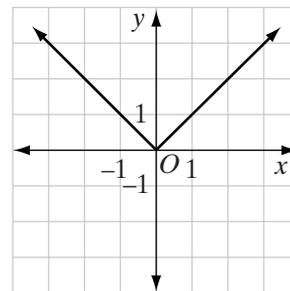
$$y = x$$

$$x \geq 0$$



$$y = -x$$

$$x < 0$$



$$y = |x|$$

$$x < 0 \quad x \geq 0$$

It should be noted that for all values of x , $y = -|x|$ results in a negative value for y . Therefore, for positive values of x , the graph of $y = -|x|$ is the same as the graph of $y = -x$. For negative values of x , the graph of $y = -|x|$ is the same as the graph of $y = x$.

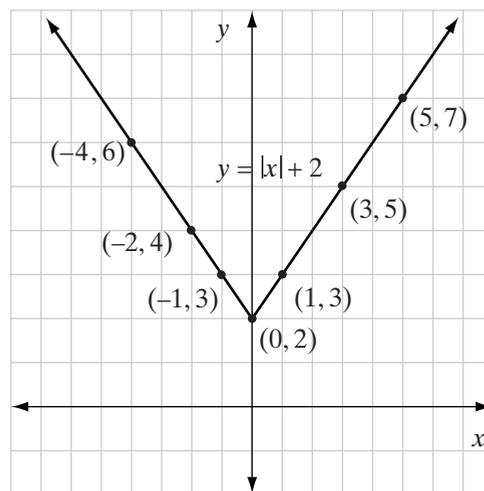
EXAMPLE 1

Draw the graph of $y = |x| + 2$.

Solution (1) Make a table of values:

x	$ x + 2$	y
-4	$ -4 + 2$	6
-2	$ -2 + 2$	4
-1	$ -1 + 2$	3
0	$ 0 + 2$	2
1	$ 1 + 2$	3
3	$ 3 + 2$	5
5	$ 5 + 2$	7

(2) Plot the points and draw rays to connect the points that were graphed:



Calculator Solution (1) Enter the equation into Y_1 :

ENTER: **Y=** **MATH** **↓** **ENTER**

X,T,θ,n **)** **+** **2**

DISPLAY:

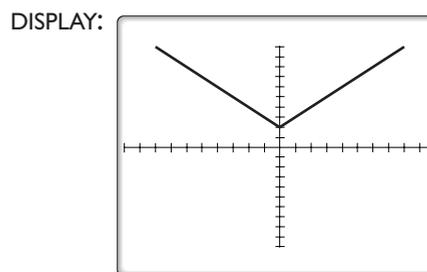
```

PLOT1 PLOT2 PLOT3
\Y1=ABS(X)+2

```

(2) Graph to the standard window:

ENTER: **ZOOM** **6**

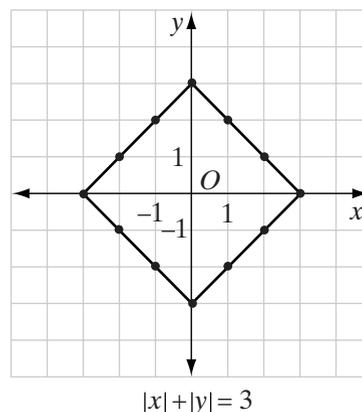
**EXAMPLE 2**

Draw the graph of $|x| + |y| = 3$.

Solution By the definition of absolute value, $|x| = |-x|$ and $|y| = |-y|$.

- Since $(1, 2)$ is a solution, $(-1, 2)$, $(-1, -2)$ and $(1, -2)$ are solutions.
- Since $(2, 1)$ is a solution, $(-2, 1)$, $(-2, -1)$ and $(2, -1)$ are solutions.
- Since $(0, 3)$ is a solution, $(0, -3)$ is a solution.
- Since $(3, 0)$ is a solution, $(-3, 0)$ is a solution.

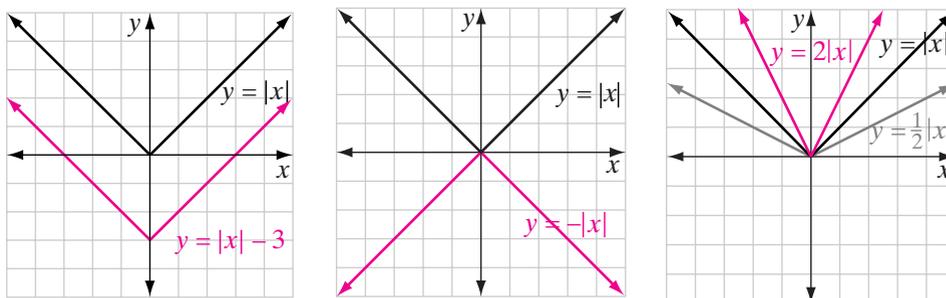
Plot the points that are solutions, and draw the line segments joining them.



Translating, Reflecting, and Scaling Graphs of Absolute Value Functions

Just as linear functions can be translated, reflected, or scaled, graphs of absolute value functions can also be manipulated by working with the graph of the absolute value function $y = |x|$.

For instance, the graph of $y = |x| - 5$ is the graph of $y = |x|$ shifted 5 units down. The graph of $y = -|x|$ is the graph of $y = |x|$ reflected in the x -axis. The graph of $y = 2|x|$ is the graph of $y = |x|$ stretched vertically by a factor of 2, while the graph of $y = \frac{1}{2}|x|$ is the graph of $y = |x|$ compressed vertically by a factor of $\frac{1}{2}$.



Translation Rules for Absolute Value Functions

If c is *positive*:

- ▶ The graph of $y = |x| + c$ is the graph of $y = |x|$ shifted c units up.
- ▶ The graph of $y = |x| - c$ is the graph of $y = |x|$ shifted c units down.

For absolute value functions, there are two additional translations that can be done to the graph of $y = |x|$, *horizontal* shifting to left or to the right.

If c is positive:

- ▶ The graph of $y = |x + c|$ is the graph of $y = |x|$ shifted c units to the left.
- ▶ The graph of $y = |x - c|$ is the graph of $y = |x|$ shifted c units to the right.

Reflection Rule for Absolute Value Functions

- ▶ The graph of $y = -|x|$ is the graph of $y = |x|$ reflected across the x -axis.

Scaling Rules for Absolute Value Functions

- ▶ When $c > 1$, the graph of $y = c|x|$ is the graph of $y = |x|$ stretched vertically by a factor of c .
- ▶ When $0 < c < 1$, the graph of $y = c|x|$ is the graph of $y = |x|$ compressed vertically by a factor of c .

EXAMPLE 3

In **a–e**, write an equation for the resulting function if the graph of $y = |x|$ is:

- a. shifted 2.5 units down
- b. shifted 6 units to the right
- c. stretched vertically by a factor of 3 and shifted 5 units up
- d. compressed vertically by a factor of $\frac{1}{3}$ and reflected in the x -axis
- e. reflected in the x -axis, shifted 1 unit up, and shifted 1 unit to the left

Solution a. $y = |x| - 2.5$ *Answer*

b. $y = |x - 6|$ *Answer*

c. First, stretch vertically by a factor of 3:

$$y = 3|x|$$

Then, translate the resulting function 5 units up:

$$y = 3|x| + 5 \text{ *Answer*}$$

d. First, compress vertically by a factor of $\frac{1}{3}$:

$$y = \frac{1}{3}|x|$$

Then, reflect in the x -axis:

$$y = -\frac{1}{3}|x| \text{ *Answer*}$$

e. First, reflect in the x -axis:

$$y = -|x|$$

Then, translate the resulting function 1 unit up:

$$y = -|x| + 1$$

Finally, translate the resulting function 1 unit to the left:

$$y = -|x + 1| + 1 \text{ *Answer*}$$



EXERCISES

Writing About Mathematics

- Charity said that the graph of $y = |2x| + 1$ is the graph of $y = 2x + 1$ for $x \geq 0$ and the graph of $-2x + 1$ for $x < 0$. Do you agree with Charity? Explain why or why not.
- April said that $|x| + |y| = 5$ is a function. Do you agree with April? Explain why or why not.
- Euclid said that, for positive values of c , the graph of $y = |cx|$ is the same as the graph of $y = c|x|$. Do you agree with Euclid? If so, prove Euclid's statement.

Developing Skills

In 4–15, graph each equation.

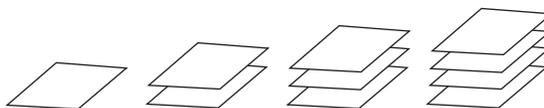
- | | | | |
|------------------|---------------------|---|---------------------------|
| 4. $y = x - 1$ | 5. $y = x + 3$ | 6. $y = x - 1 $ | 7. $y = x + 3 $ |
| 8. $y = 2 x $ | 9. $y = 2 x + 1$ | 10. $ x + y = 5$ | 11. $ x + 2 y = 7$ |
| 12. $ y = x $ | 13. $ x + y = 4$ | 14. $\frac{ x }{2} + \frac{ y }{4} = 1$ | 15. $2 x + 4 y - 6 = 0$ |

In 16–19, describe the translation, reflection, and/or scaling that must be applied to $y = |x|$ to obtain the graph of each given function.

- | | | | |
|--------------------|---------------------|-----------------------|--------------------------|
| 16. $y = - x - 4$ | 17. $y = -2 x + 2$ | 18. $y = x + 2 - 3$ | 19. $y = - x - 1.5 + 4$ |
|--------------------|---------------------|-----------------------|--------------------------|

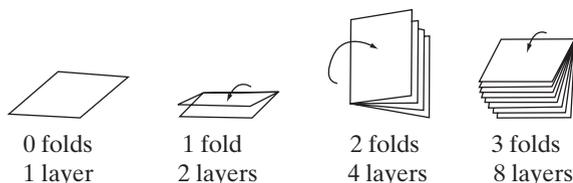
9-1 I GRAPHS INVOLVING EXPONENTIAL FUNCTIONS

A piece of paper is one layer thick. If we place another piece of paper on top of it, the stack is two layers thick. If we place another piece of paper on top, the stack is now three layers thick.



As the process continues, we can describe the number of layers, y , in terms of the number of sheets added, x , by the linear function $y = 1 + x$. For example, after we have added seven pieces of paper, the stack is eight layers thick. This is an example of **linear growth** because the change can be described by a linear function.

Now consider another experiment. A piece of paper is one layer thick. If the paper is folded in half, the stack is two layers thick. If the stack is folded again, the stack is four layers thick. If the stack is folded a third time, it is eight layers thick.



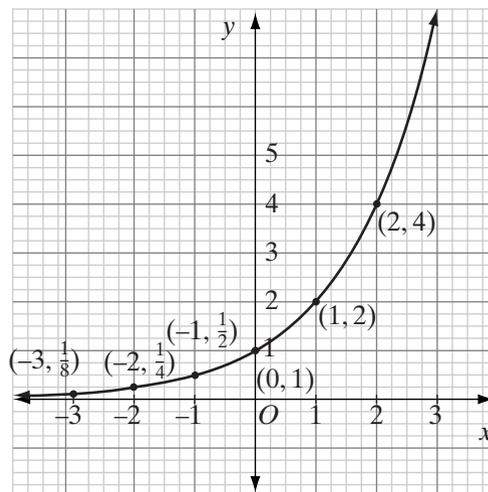
Although we will reach a point at which it is impossible to fold the paper, imagine that this process can continue. We can describe the number of layers of paper, y , in terms of the number of folds, x , by an **exponential function**, $y = 2^x$. The table at the right shows some values for this function. Such functions are said to be **nonlinear**.

After five folds, the stack is 32 layers thick. After ten folds, the stack would be 2^{10} or 1,024 layers thick. In other words, it would be thicker than this book. This is an example of **exponential growth** because the growth is described by an exponential function $y = b^x$. When the base b is a positive number greater than 1 ($b > 1$), y increases as x increases.

We can draw the graph of the exponential function, adding zero and negative integral values to those given above. Locate the points on the coordinate plane and draw a smooth curve through them. When we draw the curve, we are assuming that the domain of the independent variable, x , is the set of real numbers. You will learn about powers that have exponents that are not integers in more advanced math courses.

x	2^x	$y = 2^x$
0	2^0	1
1	2^1	2
2	2^2	4
3	2^3	8
4	2^4	16
5	2^5	32

x	2^x	$y = 2^x$
0	2^0	1
-1	2^{-1}	$\frac{1}{2}$
-2	2^{-2}	$\frac{1}{4}$
-3	2^{-3}	$\frac{1}{8}$



There are many examples of exponential growth in the world around us. Over an interval, certain populations—for example of bacteria, or rabbits, or people—grow exponentially.

Compound interest is also an example of exponential growth. If a sum of money, called the principal, P , is invested at 4% interest, then after one year the value of the investment, A , is the principal plus the interest. Recall that $I = Prt$. In this case, $r = 0.04$ and $t = 1$.

$$A = P + I = P + Prt = P(1 + rt) = P(1 + 0.04(1)) = P(1.04)$$

After 2 years, the new principal is $(1.04)^2P$. Replace P by $1.04P$ in the formula $A = P(1.04)$:

$$A = 1.04P(1.04) = P(1.04)^2$$

After 3 years, the new principal is $(1.04)^3P$. Replace P by $P(1.04)^2$ in the formula $A = P(1.04)$:

$$A = (1.04)^2P(1.04) = P(1.04)^3$$

In general, after n years, the value of an investment for which the rate of interest is r can be found by using the formula

$$A = P(1 + r)^n.$$

Note that in this formula for exponential growth, P is the amount present at the beginning; r , a positive number, is the rate of growth; and n is the number of intervals of time during which the growth has been taking place. Since r is a positive number, the base, $1 + r$, is a number greater than 1.

Compare the formula $A = P(1 + r)^n$ with the exponential function $y = 2^x$ used to determine the number of layers in a stack after x folds. $P = 1$ since we started with 1 layer. The base, $(1 + r) = 2$, so $r = 1$ or 100%. Doubling the number of layers in the stack is an increase equal to the size of the stack, that is, an increase of 100%.

An exponential change can be a decrease as well as an increase. Consider this example. Start with a large piece of paper. If we cut the paper into two equal parts, each part is one-half of the original piece. Now if we cut one of these pieces into two equal parts, each of the parts is one-fourth of the original piece. As this process continues, we can describe the part of the original piece that results from each cut in terms of the number of cuts. The function $y = \left(\frac{1}{2}\right)^x$ gives the part of the original piece, y , that is the result of x cuts. This is an example of **exponential decay**. For the exponential function $y = b^x$, when the base b is a positive number less than 1 ($0 < b < 1$), y decreases as x increases.

x	$\left(\frac{1}{2}\right)^x$	y
0	$\left(\frac{1}{2}\right)^0$	1
1	$\left(\frac{1}{2}\right)^1$	$\frac{1}{2}$
2	$\left(\frac{1}{2}\right)^2$	$\frac{1}{4}$
3	$\left(\frac{1}{2}\right)^3$	$\frac{1}{8}$
4	$\left(\frac{1}{2}\right)^4$	$\frac{1}{16}$
5	$\left(\frac{1}{2}\right)^5$	$\frac{1}{32}$

Some examples of exponential decay include a decrease in population, a fund, or the value of a machine when that decrease can be represented by a constant rate at regular intervals. The radioactive decay of a chemical element such as carbon is also an example of exponential decay.

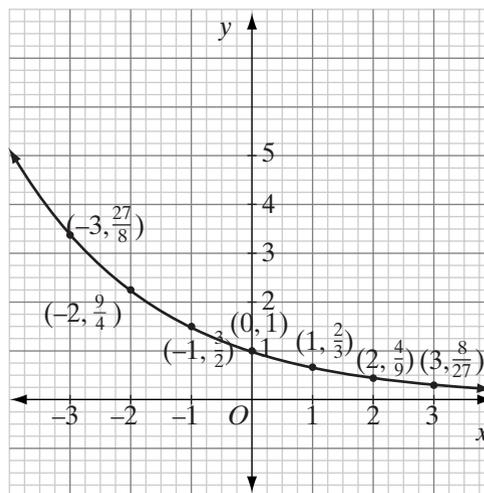
Problems of exponential decay can be solved using the same formula as that for exponential growth. In exponential decay, the rate of change is a negative number between -1 and 0 so that the base is a number between 0 and 1 . This is illustrated in Example 3.

EXAMPLE 1

Draw the graph of $y = \left(\frac{2}{3}\right)^x$.

Solution Make a table of values from $x = -3$ to $x = 3$. Plot the points and draw a smooth curve through them.

x	$\left(\frac{2}{3}\right)^x$	y
-3	$\left(\frac{2}{3}\right)^{-3} = \left(\frac{3}{2}\right)^3$	$\frac{27}{8}$
-2	$\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2$	$\frac{9}{4}$
-1	$\left(\frac{2}{3}\right)^{-1} = \left(\frac{3}{2}\right)^1$	$\frac{3}{2}$
0	$\left(\frac{2}{3}\right)^0$	1
1	$\left(\frac{2}{3}\right)^1$	$\frac{2}{3}$
2	$\left(\frac{2}{3}\right)^2$	$\frac{4}{9}$
3	$\left(\frac{2}{3}\right)^3$	$\frac{8}{27}$



Calculator Solution (1) Enter the equation into Y_1 :

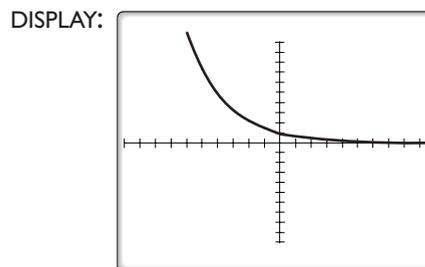
ENTER: $Y=$ (2 \div 3)

\wedge X,T, θ ,n

DISPLAY: `PLOT1 PLOT2 PLOT3`
`\Y1 = (2/3)^X`

(2) Graph to the standard window:

ENTER: **ZOOM** 6



EXAMPLE 2

A bank is advertising a rate of 5% interest compounded annually. If \$2,000 is invested in an account at that rate, find the amount of money in the account after 10 years.

Solution This is an example of exponential growth. Use the formula $A = P(1 + r)^n$ with $r = 0.05$, $P =$ the initial investment, and $n = 10$ years.

$$A = 2,000(1 + 0.05)^{10} = 2,000(1.05)^{10} = 3,257.78954$$

Answer The value of the investment is \$3,257.79 after 10 years. ■

EXAMPLE 3

The population of a town is decreasing at the rate of 2.5% per year. If the population in the year 2000 was 28,000, what will be the expected population in 2015 if this rate of decrease continues? Give your answer to the nearest thousand.

Solution Use the formula for exponential growth or decay. The rate of decrease is entered as a negative number so that the base is a number between 0 and 1.

$$r = -0.025$$

$P =$ the initial population

$n = 15$ years ($x = 0$ corresponds to the year 2000)

$$A = P(1 + r)^n = 28,000(1 + (-0.025))^{15} = 28,000(0.975)^{15} = 19,152.5792$$

Answer The population will be about 19,000 in 2015. ■

EXERCISES**Writing About Mathematics**

1. The equation $y = b^x$ is an exponential function. If the graph of the function is a smooth curve, explain why the value of b cannot be negative.
2. The equation $A = P(1 + r)^n$ can be used for both exponential growth and exponential decay when r represents the percent of increase or decrease.
 - a. How does the value of r that is used in this equation for exponential growth differ from that of exponential decay?
 - b. If the equation represents exponential growth, can the rate be greater than 100%? Explain why or why not.
 - c. If the equation represents exponential decay, can the rate be greater than 100%? Explain why or why not.

3. Euler says that the graph of $y = 3^{x+2}$ is the graph of $y = 3^x$ shifted 2 units to the left and that the graph of $y = 3^x + 2$ is the graph of $y = 3^x$ shifted 2 units up. Do you agree with Euler? Explain why or why not.

Developing Skills

In 4–9, sketch each graph using as values of x the integers in the given interval.

4. $y = 3^x$, $[-3, 3]$ 5. $y = 4^x$, $[-2, 2]$ 6. $y = 1.5^x$, $[-4, 4]$
 7. $y = \left(\frac{2}{5}\right)^x$, $[-3, 3]$ 8. $y = \left(\frac{1}{3}\right)^x$, $[-2, 2]$ 9. $y = 1.25^x$, $[0, 10]$

10. Compare each of the exponential graphs drawn in exercises 4 through 9.
- What point is common to all of the graphs?
 - If the value of the base is greater than 1, in what direction does the graph curve?
 - If the value of the base is between 0 and 1, in what direction does the graph curve?
11. a. Sketch the graph of $y = \left(\frac{1}{2}\right)^x$ in the interval $[-3, 3]$.
 b. Sketch the graph of $y = 2^{-x}$ in the interval $[-3, 3]$.
 c. What do you observe about the graphs drawn in **a** and **b**?

Applying Skills

12. In 2005, the population of a city was 25,000. The population increased by 20% in each of the next three years. If this rate of increase continues, what will be the population of the city in 2012?
13. Alberto invested \$5,000 at 6% interest compounded annually. What will be the value of Alberto's investment after 8 years?
14. Mrs. Boyko has a trust fund from which she withdraws 5% each year. If the fund has a value of \$50,000 this year, what will be the value of the fund after 10 years?
15. Hailey has begun a fitness program. The first week she ran 1 mile every day. Each week she increases the amount that she runs each day by 20%. In week 10, how many miles does she run each day? Give your answer to the nearest mile.
16. Alex received \$75 for his birthday. In the first week after his birthday, he spent one-third of the money. In the second week and each of the following weeks, he spent one-third of the money he had left. How much money will Alex have left after 5 weeks?
17. During your summer vacation, you are offered a job at which you can work as many days as you choose. If you work 1 day, you will be paid \$0.01. If you work 2 days you will be paid a total of \$0.02. If you work 3 days you will be paid a total of \$0.04. If you continue to work, your total pay will continue to double each day.
- Would you accept this job if you planned to work 10 days? Explain why or why not.
 - Would you accept this job if you planned to work 25 days? Explain why or why not.

CHAPTER SUMMARY

The solutions of equations or inequalities in two variables are ordered pairs of numbers. The set of points whose coordinates make an equation or inequality true is the graph of that equation or inequality. The **graph of a linear function** of the form $Ax + By = C$ is a straight line.

The **domain of a function** is the set of all values the **independent variable**, the x -coordinate, is allowed to take. This determines the **range of a function**, that is, the set of all values the **dependent variable**, the y -coordinate, will take.

Set-builder notation provides a mathematically concise method of describing the elements of a set. **Roster form** lists every element of a set exactly once.

A linear function can be written in the form $y = mx + b$, where m is the slope and b is the y -intercept of the line that is the graph of the function. A line parallel to the x -axis has a slope of 0, and a line parallel to the y -axis has no slope. The **slope** of a line is the ratio of the change in the vertical direction to the change in the horizontal direction. If (x_1, y_1) and (x_2, y_2) are two points on a line, the slope of the line is

$$\text{slope} = m = \frac{y_2 - y_1}{x_2 - x_1}$$

If y varies directly as x , the ratio of y to x is a constant. **Direct variation** can be represented by a line through the origin whose slope is the **constant of variation**.

The graph of $y = mx + b$ separates the plane into two **half-planes**. The half-plane above the graph of $y = mx + b$ is the graph of $y > mx + b$, and the half-plane below the graph of $y = mx + b$ is the graph of $y < mx + b$.

The graph of $y = |x|$ is the **union** of the graph of $y = -x$ for $x < 0$ and the graph of $y = x$ for $x \geq 0$.

The equation $y = b^x$, $b > 0$ and $b \neq 1$, is an example of an exponential function. The exponential equation $A = P(1 + r)^n$ is a formula for exponential growth or decay. A is the amount present after n intervals of time, P is the amount present at time 0, and r is the rate of increase or decrease. In exponential growth, r is a positive number. In exponential decay, r is a negative number in the interval $(-1, 0)$.

The function $y = x$ and $y = |x|$ can be translated, reflected, or scaled to graph other linear and absolute value functions.

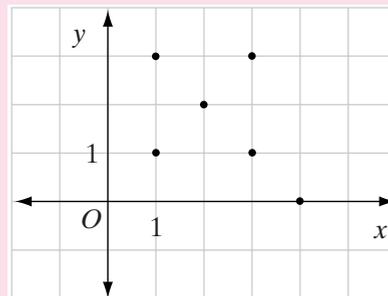
	Linear Function	Absolute Value
Vertical translations ($c > 0$)	$y = x + c$ (up)	$y = x + c$ (up)
	$y = x - c$ (down)	$y = x - c$ (down)
Horizontal translations ($c > 0$)	$y = x + c$ (left)	$y = x + c $ (left)
	$y = x - c$ (right)	$y = x - c $ (right)
Reflection across the x-axis	$y = -x$	$y = - x $
Vertical stretching ($c > 1$)	$y = cx$	$y = c x $
Vertical compression ($0 < c < 1$)	$y = cx$	$y = cx$

VOCABULARY

- 9-1** Roster form • Set-builder notation • Member of a set (\in) • Not a member of a set (\notin) • Relation • Domain of a relation • Range of a relation • Function • Domain of a function • Range of a function • Independent variable • Dependent variable
- 9-2** Graph • Standard form • Linear equation
- 9-4** Slope • Unit rate of change
- 9-6** y -intercept • x -intercept • Intercept form a linear equation • Slope-intercept form of a linear equation
- 9-8** Equation of a direct variation
- 9-9** Plane divider • Half-plane
- 9-11** Linear growth • Exponential function • Nonlinear functions • Exponential growth • Exponential decay

REVIEW EXERCISES

- 1.** Determine **a.** the domain and **b.** the range of the relation shown in the graph to the right. **c.** Is the relation a function? Explain why or why not.



- 2.** Draw the graph of $|x| + |y| = 5$. Is the graph of $|x| + |y| = 5$ a square? Prove your answer.
- 3.** A function is a set of ordered pairs in which no two ordered pairs have the same first element. Explain why $\{(x, y) \mid y < 2x + 1\}$ is not a function.
- 4.** What is the slope of the graph of $y = -2x + 5$?
- 5.** What is the slope of the line whose equation is $3x - 2y = 12$?
- 6.** What are the intercepts of the line whose equation is $3x - 2y = 12$?
- 7.** What is the slope of the line that passes through points $(4, 5)$ and $(6, 1)$?

In 8–13, in each case: **a.** Graph the equation or inequality. **b.** Find the domain and range of the equation or inequality.

8. $y = -x + 2$

9. $y = 3$

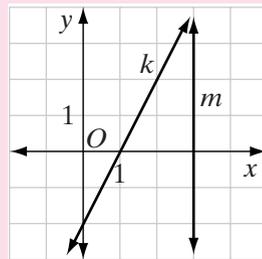
10. $y = \frac{2}{3}x$

11. $x + 2y = 8$

12. $y - x > 2$

13. $2x - y \geq 4$

In 14–22, refer to the coordinate graph to answer each question.

14. What is the slope of line k ?15. What is the x -intercept of line k ?16. What is the y -intercept of line k ?17. What is the slope of a line that is parallel to line k ?18. What is the slope of a line that is perpendicular to line k ?19. What is the slope of line m ?20. What is the x -intercept of line m ?21. What is the y -intercept of line m ?22. What is the slope of a line that is perpendicular to line m ?23. If point $(d, 3)$ lies on the graph of $3x - y = 9$, what is the value of d ?

In 24–33, in each case select the numeral preceding the correct answer.

24. Which set of ordered pairs represents a function?

(1) $\{(-6, 1), (-7, 1), (1, -7), (1, -6)\}$

(2) $\{(0, 13), (1, 13), (6, 13), (112, 13)\}$

(3) $\{(3, -3), (3, -4), (-3, -4), (-3, -3)\}$

(4) $\{(1, 12), (11, 1), (112, 11), (11, 112)\}$

25. Which point does not lie on the graph of $3x - y = 9$?

(1) $(1, -6)$

(2) $(2, 3)$

(3) $(3, 0)$

(4) $(0, -9)$

26. Which ordered pair is in the solution set of $y < 2x - 4$?

(1) $(0, -5)$

(2) $(2, 0)$

(3) $(3, 3)$

(4) $(0, 2)$

27. Which equation has a graph parallel to the graph of $y = 5x - 2$?

(1) $y = -5x$

(2) $y = 5x + 3$

(3) $y = -2x$

(4) $y = 2x - 5$

28. The graph of $2x + y = 8$ intersects the x -axis at point

(1) $(0, 8)$

(2) $(8, 0)$

(3) $(0, 4)$

(4) $(4, 0)$

29. What is the slope of the graph of the equation $y = 4$?

(1) The line has no slope.

(2) 0

(3) -4

(4) 4

30. In which ordered pair is the x -coordinate 3 more than the y -coordinate?

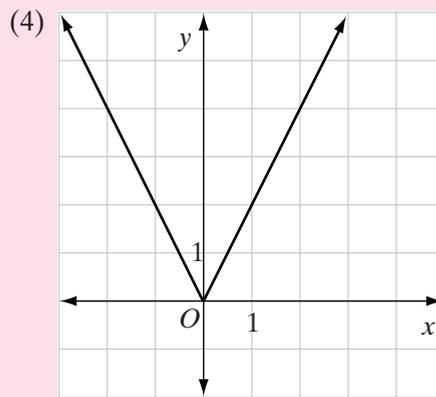
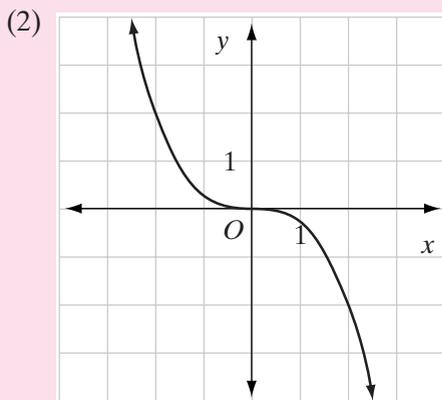
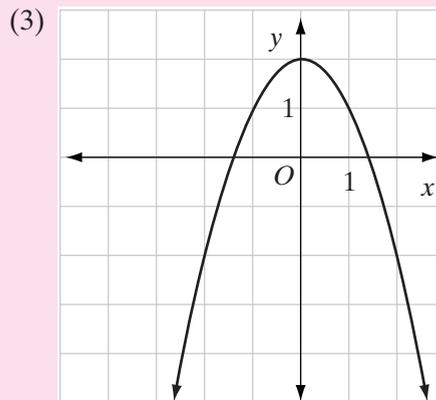
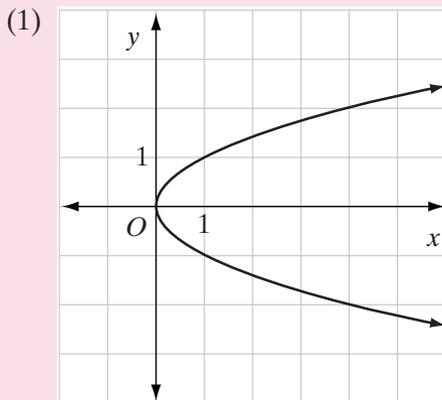
(1) $(1, 4)$

(2) $(1, 3)$

(3) $(3, 1)$

(4) $(4, 1)$

31. Which of the following is *not* a graph of a function?



32. Using the domain $D = \{4x \mid x \in \text{the set of integers and } 10 \leq 4x \leq 20\}$, what is the range of the function $y = |2 - x| + 2$?

(1) $\{-10, -14, -18\}$

(3) $\{10, 14, 16, 20\}$

(2) $\{12, 16, 20\}$

(4) $\{4, 14, 24, 34\}$

33. Which equation best describes the graph of $y = |x|$ reflected across the x -axis, shifted 9 units up, and shifted 2 units to the left?

(1) $y = |-x + 2| + 9$

(3) $y = |x - 2| + 9$

(2) $y = -|x + 2| + 9$

(4) $y = -|x - 2| - 9$

34. a. Plot points $A(5, -2)$, $B(3, 3)$, $C(-3, 3)$, and $D(-5, -2)$.

b. Draw polygon $ABCD$.

c. What kind of polygon is $ABCD$?

d. Find DA and BC .

e. Find the length of the altitude from C to \overline{DA} .

f. Find the area of $ABCD$.

In 35–38, graph each equation.

35. $y = |x - 2|$ 36. $y = |x| - 2$ 37. $|x| + 2|y| = 6$ 38. $y = 2.5^x$

39. Tiny Tot Day Care Center has changed its rates. It now charges \$350 a week for children who stay at the center between 8:00 A.M. and 5:00 P.M. The center charges an additional \$2.00 for each day that the child is not picked up by 5:00 P.M.
- Write an equation for the cost of day care for the week, y , in terms of x , the number of days that the child stays beyond 5:00 P.M.
 - Define the slope of the equation found in part **a** in terms of the information provided.
 - What is the charge for 1 week for a child who was picked up at the following times: Monday at 5:00, Tuesday at 5:10, Wednesday at 6:00, Thursday at 5:00, and Friday at 5:25?
40. Each time Raphael put gasoline into his car, he recorded the number of gallons of gas he needed to fill the tank and the number of miles driven since the last fill-up. The chart below shows his record for one month.

Gallons of Gasoline	12	8.5	10	4.5	13
Number of Miles	370	260	310	140	420

- Plot points on a graph to represent the information in the chart. Let the horizontal axis represent the number of gallons of gasoline and the vertical axis the number of miles driven.
 - Find the average number of gallons of gasoline per fill-up.
 - Find the average number of miles driven between fill-ups.
 - Locate a point on the graph whose x -coordinate represents the average number of gallons of gasoline per fill-up and whose y -coordinate is the average number of miles driven between fill-ups.
 - It is reasonable that $(0, 0)$ is a point on the graph? Draw, through the point that you plotted in **d** and the point $(0, 0)$, a line that could represent the information in the chart.
 - Raphael drove 200 miles since his last fill-up. How many gallons of gasoline should he expect to need to fill the tank based on the line drawn in **e**?
41. Mandy and Jim are standing 20 feet apart. Each second, they decrease the distance between each other by one-half.
- Write an equation to show their distance apart, D , at s seconds.
 - How far apart will they be after 5 seconds?
 - According to your equation, will Mandy and Jim ever meet?

42. According to the curator of zoology at the Rochester Museum and Science Center, if you count the number of chirps of a tree cricket in 15 seconds and add 40, you will have a close approximation of the actual air temperature in degrees Fahrenheit. To test this statement, Alexa recorded, on several summer evenings, the temperature in degrees Fahrenheit and the number of chirps of a tree cricket in 1 minute. She obtained the following results:

Chirps/minute	150	170	140	125	108	118	145	68	95	110
Temperature	80	85	76	71	65	70	76	56	63	67

- Write an equation that states the relationship between the number of chirps per minute of the tree cricket, c , to the temperature in degrees Fahrenheit, F . (To change chirps per minute to chirps in 15 seconds, divide c by 4.)
 - Draw a graph of the data given in the table. Record the number of chirps per minute on the horizontal axis, and the temperature on the vertical axis.
 - Draw the graph of the equation that you wrote in part **a** on the same set of axes that you used for part **b**.
 - Can the data that Alexa recorded be represented by the equation that you wrote for part **a**? Explain your answer.
43. In order to control the deer population in a local park, an environmental group plans to reduce the number of deer by 5% each year. If the deer population is now estimated to be 4,000, how many deer will there be after 8 years?

Exploration

- STEP 1.** Draw the graph of $y = x - 2$.
- STEP 2.** Let A be the point at which the graph intersects the x -axis, B be any point on the line that has an x -coordinate greater than A , and C be the point at which a vertical line from B intersects the x -axis.
- STEP 3.** Compare the slope of $y = x - 2$ with $\tan \angle BAC$.
- STEP 4.** Use a calculator to find $m\angle BAC$.
- STEP 5.** Repeat Steps 2 through 4 for three other lines that have a positive slope.
- STEP 6.** Draw the graph of $y = -x + 2$ and repeat Step 2. Let the measure of an acute angle between the x -axis and a line that slants upward be positive and the measure of an acute angle between the x -axis and a line that slants downward be negative.
- STEP 7.** Repeat Steps 3 and 4 for $y = -x + 2$.
- STEP 8.** Repeat Steps 2 through 4 for three other lines that have a negative slope.
- STEP 9.** What conclusion can you draw?

CUMULATIVE REVIEW

CHAPTERS 1–9

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

- The product of $-5a^3$ and $-3a^4$ is
 - $15a^7$
 - $15a^{12}$
 - $-15a^7$
 - $-15a^{12}$
- A basketball team won b games and lost 4. The ratio of games won to games played is
 - $\frac{b}{4}$
 - $\frac{b}{b+4}$
 - $\frac{4}{b}$
 - $\frac{4}{b+4}$
- In decimal notation, 8.72×10^{-2} is
 - 87,200
 - 872
 - 0.0872
 - 0.00872
- Which equation is not an example of direct variation?
 - $y = 2x$
 - $\frac{y}{x} = 2$
 - $y = \frac{2}{x}$
 - $y = \frac{x}{2}$
- The graph of $y - 2x = 4$ is parallel to the graph of
 - $y = 2x + 5$
 - $2x + y = 7$
 - $y = -2x + 3$
 - $2x + y = 0$
- The area of a triangle is 48.6 square centimeters. The length of the base of the triangle is 3.00 centimeters. The length of the altitude of the triangle is
 - 32.4 centimeters
 - 16.2 centimeters
 - 8.10 centimeters
 - 3.24 centimeters
- The measure of the larger acute angle of a right triangle is 15 more than twice the measure of the smaller. What is the measure of the larger acute angle?
 - 25
 - 37.5
 - 55
 - 65
- Solve for x : $1 - 2(x - 4) = x$.
 - $-\frac{1}{2}$
 - 2
 - 3
 - 4.5
- The measure of one leg of a right triangle is 9 and the measure of the hypotenuse is 41. The measure of the other leg is
 - 32
 - 40
 - 41.98
 - $\sqrt{1,762}$
- Which of the following geometric figures does not always have a pair of congruent angles?
 - a parallelogram
 - an isosceles triangle
 - a rhombus
 - a trapezoid

Part II

Answer all questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit.

11. Cory has planted a rectangular garden. The ratio of the length to the width of the garden is $5 : 7$. Cory bought 100 feet of fencing. After enclosing the garden with the fence, he still had 4 feet of fence left. What are the dimensions of his garden?
12. A square, $ABCD$, has a vertex at $A(4, 2)$. Side \overline{AB} is parallel to the x -axis and $AB = 7$. What could be the coordinates of the other three vertices? Explain how you know that for the coordinates you selected $ABCD$ is a square.

Part III

Answer all questions in this part. Each correct answer will receive 3 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit.

13. A straight line with a slope of -2 contains the point $(2, 4)$. Find the y -coordinate of a point on this line whose x -coordinate is 5.
14. Mrs. Gantrish paid \$42 for 8 boxes of file folders. She bought some on sale for \$4 a box and the rest at a later time for \$6 a box. How many boxes of file folders did she buy on sale?

Part IV

Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit.

15. Concentric circles (circles that have the same center) are used to divide the circular face of a dartboard into 4 regions of equal area. If the radius of the board is 12.0 inches, what is the radius of each of the concentric circles? Express your answers to the *nearest tenth of an inch*.
16. A ramp is to be built from the ground to a doorway that is 4.5 feet above the ground. What will be the length of the ramp if it makes an angle of 22° with the ground? Express your answer to the *nearest foot*.