CHAPTER

3

ALGEBRAIC EXPRESSIONS AND OPEN SENTENCES

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An express delivery company will deliver a letter or package locally, within two hours. The company has the following schedule of rates. In addition to the basic charge of $25, the cost is $3 per mile or part of a mile for the first 10 miles or less and $4.50 per mile or part of a mile for each additional mile over 10.

Costs such as those described, that vary according to a schedule, are often shown by a formula or set of formulas. Formulas can be used to solve many different problems.

In this chapter, you will learn to write algebraic expressions and formulas, to use algebraic expressions and formulas to solve problems, and to determine the solution set of an open sentence.

3-1 USING LETTERS TO REPRESENT NUMBERS

Eggs are usually sold by the dozen, that is, 12 in a carton. Therefore, we know that:

In 1 carton, there are 12×1 or 12 eggs.
In 2 cartons, there are 12×2 or 24 eggs.
In 3 cartons, there are 12×3 or 36 eggs.
In n cartons, there are $12 \times n$ or $12n$ eggs.

Here, n is called a **variable** or a **placeholder** that can represent different numbers from the set of whole numbers, $\{1, 2, 3, \ldots\}$. The set of numbers that can replace a variable is called the **domain** or the **replacement set** of that variable.

Recall that a numerical expression contains only numbers. An **algebraic expression**, such as $12n$, however, is an expression or phrase that contains one or more variables.

In this section, you will see how verbal phrases are translated into algebraic expressions, using letters as variables and using symbols to represent operations.

Verbal Phrases Involving Addition

The algebraic expression $a + b$ may be used to represent several different verbal phrases, such as:

a plus b	a added to b	a is increased by b
the sum of a and b	b is added to a	b more than a

The word *exceeds* means "is more than." Thus, the number that exceeds 5 by 2 can be written as "2 *more than* 5" or $5 + 2$.

Compare the numerical and algebraic expressions shown below.

A numerical expression:	The number that exceeds 5 by 2 is $5 + 2$, or 7.
An algebraic expression:	The number that exceeds a by b is $a + b$.

Verbal Phrases Involving Subtraction

The algebraic expression $a - b$ may be used to represent several different verbal phrases, such as:

a minus b	a decreased by b	b less than a
b subtracted from a	a diminished by b	a reduced by b

the difference between a and b

Verbal Phrases Involving Multiplication

The algebraic expressions $a \times b$, $a \cdot b$, $(a)(b)$ and ab may be used to represent several different verbal phrases, such as:

a times b	the product of a and b	b multiplied by a

The preferred form to indicate multiplication in algebra is ab. Here, a product is indicated by using *no symbol* between the variables being multiplied.

The multiplication symbol $\times$ is avoided in algebra because it can be confused with the letter or variable x. The raised dot, which is sometimes mistaken for a decimal point, is also avoided. Parentheses are used to write numerical expressions: (3)(5)(2) or 3(5)(2). Note that all but the first number must be in parentheses. In algebraic expressions, parentheses may be used but they are not needed: $3(b)(h) = 3bh$.

Verbal Phrases Involving Division

The algebraic expressions $a \div b$ and $\frac{a}{b}$ may be used to represent several different verbal phrases, such as:

a divided by *b*   the quotient of *a* and *b*

The symbols $a \div 4$ and $\frac{a}{4}$ mean one-fourth of *a* as well as *a* divided by 4.

Phrases and Commas

In some verbal phrases, using a comma can prevent misreading. For example, in "the product of x and y, decreased by 2," the comma after y makes it clear that the x and y are to be multiplied before subtracting 2 and can be written as $(xy) - 2$ or $xy - 2$. Without the comma, the phrase, "the product of x and y decreased by 2," would be written $x(y - 2)$.

EXAMPLE 1

Use mathematical symbols to translate the following verbal phrases into algebraic language:

	Answers
a. w more than 3	$3 + w$
b. w less than 3	$3 - w$
c. r decreased by 2	$r - 2$
d. the product of $5r$ and s	$5rs$
e. twice x, decreased by 10	$2x - 10$
f. 25, diminished by 4 times n	$25 - 4n$
g. the sum of t and u, divided by 6	$\frac{t+u}{6}$
h. 100 decreased by twice $(x + 5)$	$100 - 2(x + 5)$

EXERCISES

Writing About Mathematics

1. Explain why the sum of a and 4 can be written as $a + 4$ or as $4 + a$.

2. Explain why 3 less than a can be written as $a - 3$ but not as $3 - a$.

Developing Skills

In 3–20, use mathematical symbols to translate the verbal phrases into algebraic language.

3. y plus 8
4. 4 minus r
5. 7 times x
6. x times 7
7. x divided by 10
8. 10 divided by x
9. c decreased by 6
10. one-tenth of w
11. the product of x and y
12. 5 less than d
13. 8 divided by y
14. y multiplied by 10
15. t more than w
16. one-third of z
17. twice the difference of p and q
18. a number that exceeds m by 4
19. 5 times x, increased by 2
20. 10 decreased by twice a

In 21–30, using the letter n to represent "number," write each verbal phrase in algebraic language.

21. a number increased by 2
22. 20 more than a number
23. 8 increased by a number
24. a number decreased by 6
25. 2 less than a number
26. 3 times a number
27. three-fourths of a number
28. 4 times a number, increased by 3
29. 3 less than twice a number
30. 10 times a number, decreased by 2

In 31–34, use the given variable(s) to write an algebraic expression for each verbal phrase.

31. the number of baseball cards, if b cards are added to a collection of 100 cards

32. Hector's height, if he was h inches tall before he grew 2 inches

33. the total cost of n envelopes that cost $0.39 each

34. the cost of one pen, if 12 pens cost d dollars

3-2 TRANSLATING VERBAL PHRASES INTO SYMBOLS

A knowledge of arithmetic is important in algebra. Since the variables represent numbers that are familiar to you, it will be helpful to solve each problem by first using a simpler related problem; that is, relate similar arithmetic problems to the given algebraic one.

Procedure

To write an algebraic expression involving variables:

1. Think of a similar problem in arithmetic.
2. Write an expression for the arithmetic problem, using numbers.
3. Write a similar expression for the problem, using letters or variables.

EXAMPLE 1

Represent each phrase by an algebraic expression.

a. a distance that is 20 meters shorter than x meters

b. a bill for n baseball caps, each costing d dollars

c. a weight that is 40 pounds heavier than p pounds

d. an amount of money that is twice d dollars

Solution **a.** *How to Proceed*

(1) Think of a similar problem in arithmetic:	Think of a distance that is 20 meters shorter than 50 meters.
(2) Write an expression for this arithmetic problem:	$50 - 20$
(3) Write a similar expression using the letter x in place of 50.	$x - 20$ *Answer*

b. *How to Proceed*

(1) Think of a similar problem in arithmetic:	Think of a bill for 6 caps, each costing 5 dollars.
(2) Write an expression for this arithmetic problem. Multiply the number of caps by the cost of one cap:	$6(5)$
(3) Write a similar expression using n and d:	nd *Answer*

Note: After some practice, you will be able to do steps (1) and (2) mentally.

c. $(p + 40)$ pounds or $(40 + p)$ pounds

d. $2d$ dollars

Answers **a.** $(x - 20)$ meters **b.** nd dollars

c. $(p + 40)$ or $(40 + p)$ pounds **d.** $2d$ dollars

EXAMPLE 2

Brianna paid 17 dollars for batteries and film for her camera. If the batteries cost x dollars, express the cost of the film in terms of x.

Solution If Brianna had spent 5 dollars for the batteries, the amount that was left is found by subtracting the 5 dollars from the 17 dollars, $(17 - 5)$ dollars. This would have been the cost of the film. If Brianna spent x dollars for the batteries, then the difference, $(17 - x)$ dollars would have been the cost of the film.

Answer $(17 - x)$ dollars

Note: In general, if we know the sum of two quantities, then we can let x represent one of these quantities and (the sum $- x$) represent the other.

EXERCISES

Writing About Mathematics

1. a. Represent the number of pounds of grapes you can buy with d dollars if each pound costs b dollars.

b. Does the algebraic expression in part **a** always represent a whole number? Explain your answer by showing examples using numbers.

2. a. If x apples cost c cents, represent the cost of one apple.

b. If x apples cost c cents, represent the cost of n apples.

c. Do the algebraic expressions in parts **a** and **b** always represent whole numbers? Explain your answer.

Developing Skills

In 3–18, represent each answer in algebraic language, using the variable mentioned in the problem.

3. The number of kilometers traveled by a bus is represented by x. If a train traveled 200 kilometers farther than the bus, represent the number of kilometers traveled by the train.

4. Mr. Gold invested $1,000 in stocks. If he lost d dollars when he sold the stocks, represent the amount he received for them.

5. The cost of a mountain bike is 5 times the cost of a skateboard. If the skateboard costs x dollars, represent the cost of the mountain bike.

6. The length of a rectangle is represented by l. If the width of the rectangle is one-half of its length, represent its width.

7. After 12 centimeters had been cut from a piece of lumber, c centimeters were left. Represent the length of the original piece of lumber.

8. Paul and Martha saved 100 dollars. If the amount saved by Paul is represented by x, represent the amount saved by Martha.

9. A ballpoint pen sells for 39 cents. Represent the cost of x pens.

10. Represent the cost of t feet of lumber that sells for g cents a foot.

11. If Hilda weighed 45 kilograms, represent her weight after she had lost x kilograms.

12. Ronald, who weighs c pounds, is d pounds overweight. Represent the number of pounds Ronald should weigh.

13. A woman spent \$250 for jeans and a ski jacket. If she spent y dollars for the ski jacket, represent the amount she spent for the jeans.

14. A man bought an article for c dollars and sold it at a profit of \$25. Represent the amount for which he sold it.

15. The width of a rectangle is represented by w meters. Represent the length of the rectangle if it exceeds the width by 8 meters.

16. The width of a rectangle is x centimeters. Represent the length of the rectangle if it exceeds twice the width by 3 centimeters.

17. If a plane travels 550 kilometers per hour, represent the distance it will travel in h hours.

18. If a car traveled for 5 hours at an average rate of r kilometers per hour, represent the distance it traveled.

19. **a.** Represent the total number of days in w weeks and 5 days.

    b. Represent the total number of days in w weeks and d days.

Applying Skills

20. An auditorium with m rows can seat a total of c people. If each row in the auditorium has the same number of seats, represent the number of seats in one row.

21. Represent the total number of calories in x peanuts and y potato chips if each peanut contains 6 calories and each potato chip contains 14 calories.

22. The charges for a telephone call are \$0.45 for the first 3 minutes and \$0.09 for each additional minute or part of a minute. Represent the cost of a telephone call that lasts m minutes when m is greater than 3.

23. A printing shop charges a 75-cent minimum for the first 8 photocopies of a flyer. Additional copies cost 6 cents each. Represent the cost of c copies if c is greater than 8.

24. A utility company measures gas consumption by the hundred cubic feet, CCF. The company has a three-step rate schedule for gas customers. First, there is a minimum charge of \$5.00 per month for up to 3 CCF of gas used. Then, for the next 6 CCF, the charge is \$0.75 per CCF. Finally, after 9 CCF, the charge is \$0.55 per CCF. Represent the cost of g CCF of gas if g is greater than 9.

3-3 ALGEBRAIC TERMS AND VOCABULARY

Terms

A **term** is a number, a variable, or any *product* or *quotient* of numbers and variables. For example: 5, x, $4y$, $8ab$, $\frac{k^2}{5}$, and $\frac{5}{c}$ are terms.

An algebraic expression that is written as a *sum* or a *difference* has more than one term. For example, $4a + 2b - 5c$ has three terms. These terms, $4a$, $2b$, and $5c$, are separated by + and − signs.

Factors of a Term

If a term contains two or more numbers or variables, then each number, each variable, and each product of numbers and variables is called a **factor** of the term, or factor of the product. For example, the factors of $3xy$ are 1, 3, x, y, $3x$, $3y$, xy, and $3xy$. When we factor whole numbers, we write only factors that are integers.

Any factor of an algebraic term is called the **coefficient** of the remaining factor, or product of factors, of that term. For example, consider the algebraic term $3xy$:

3 is the coefficient of xy $3x$ is the coefficient of y
$3y$ is the coefficient of x xy is the coefficient of 3

When an algebraic term consists of a number and one or more variables, the number is called the **numerical coefficient** of the term. For example:

In $8y$, the numerical coefficient is 8.
In $4abc$, the numerical coefficient is 4.

When the word *coefficient* is used alone, it usually means a numerical coefficient. Also, since x names the same term as $1x$, the coefficient of x is understood to be 1. This is true of all terms that contain only variables. For example:

7 is the coefficient of b in the term $7b$.   1 is the coefficient of b in the term b.
2.25 is the coefficient of gt in the term $2.25gt$.   1 is the coefficient of gt in the term gt.

Bases, Exponents, and Powers

You learned in Chapter 2 that a *power* is the product of equal factors. A power has a base and an exponent.

The *base* is one of the equal factors of the power.

The *exponent* is the number of times the base is used as a factor. (If a term is written without an exponent, the exponent is understood to be 1.)

$4^2 = 4(4)$:	base = 4	exponent = 2	power = $4^2 = 16$
$x^3 = x(x)(x)$:	base = x	exponent = 3	power = x^3
$35m$:	base = m	exponent = 1	power = m^1 or m
$5d^2 = 5(d)(d)$:	base = d	exponent = 2	power = d^2

An exponent refers only to the number or variable that is directly to its left, as seen in the last example, where 2 refers only to the base d. To show the product $5d$ as a base (or to show any sum, difference, product, or quotient as a base), we must enclose the base in parentheses.

$(5d)^2 = (5d)(5d)$:	base = $5d$	exponent = 2	power = $(5d)^2$
$(a + 4)^2 = (a + 4)(a + 4)$:	base = $(a + 4)$	exponent = 2	power = $(a + 4)^2$

Note that $-d^4$ is not the same as $(-d)^4$.

$-d^4 = -1(d)(d)(d)(d)$ is always a negative number.
$(-d)^4 = 1(-d)(-d)(-d)(-d)$ is always a positive number since the exponent is even.

EXAMPLE 1

For each term, name the coefficient, base, and exponent.

Answers

a. $4x^5$	coefficient = 4	base = x	exponent = 5
b. $-w^8$	coefficient = –1	base = w	exponent = 8
c. $2\pi r$	coefficient = 2π	base = r	exponent = 1

Note: Remember that coefficient means *numerical coefficient*, and that 2π is a real number.

EXERCISES

Writing About Mathematics

1. Does squaring distribute over multiplication, that is, does $(ab)^2 = (a^2)(b^2)$? Write $(ab)^2$ as $(ab)(ab)$ and use the associative and commutative properties of multiplication to justify your answer.

2. Does squaring distribute over addition, that is, does $(a + b)^2 = a^2 + b^2$? Substitute values for a and b to justify your answer.

Developing Skills

In 3–6, name the factors (other than 1) of each product.

3. xy **4.** $3a$ **5.** $7mn$ **6.** $1st$

In 7–14, name, in each case, the numerical coefficient of x.

7. $8x$ **8.** $(5 + 2)x$ **9.** $\frac{1}{2}x$ **10.** x

11. $-1.4x$ **12.** $2 + 7x$ **13.** $3.4x$ **14.** $-x$

In 15–22, name, in each case, the base and exponent of the power.

15. m^2 **16.** $-s^3$ **17.** t **18.** $(-a)^4$

19. 10^6 **20.** $(5y)^4$ **21.** $(x + y)^5$ **22.** $12c^3$

In 23–29, write each expression, using exponents.

23. $b \cdot b \cdot b \cdot b \cdot b$ **24.** $\pi \cdot r \cdot r$ **25.** $a \cdot a \cdot a \cdot a \cdot b \cdot b$

26. $7 \cdot r \cdot r \cdot r \cdot s \cdot s$ **27.** $(6a)(6a)(6a)$ **28.** $(a - b)(a - b)(a - b)$

29. the fourth power of $(m + 2n)$

In 30–33, write each term as a product without using exponents.

30. r^6 **31.** $5x^4$ **32.** $4a^4b^2$ **33.** $(3y)^5$

In 34–41, name, for each given term, the coefficient, base, and exponent.

34. $-3k$ **35.** $-k^3$ **36.** πr^2 **37.** $(ax)^5$

38. $\sqrt{2}y$ **39.** $0.0004t^{12}$ **40.** $\frac{3}{2}a^4$ **41.** $(-b)^3$

Applying Skills

42. If x represents the cost of a can of soda, what could $5x$ represent?

43. If r represents the speed of a car in miles per hour, what could $3r$ represent?

44. If n represents the number of CDs that Alice has, what could $n - 5$ represent?

45. If d represents the number of days until the end of the year, what could $\frac{d}{7}$ represent?

46. If s represents the length of a side of a square, what could $4s$ represent?

47. If r represents the measure of the radius of a circle, what could $2r$ represent?

48. If w represents the number of weeks in a school year, what could $\frac{w}{4}$ represent?

49. If d represents the cost of one dozen bottles of water, what could $\frac{d}{12}$ represent?

50. If q represents the point value of one field goal, what could $7q$ represent?

3-4 WRITING ALGEBRAIC EXPRESSIONS IN WORDS

In Section 1 of this chapter, we listed the words that can be represented by each of the four basic operations. We can use these same lists to write algebraic expressions in words and to write problems that can be represented by a given algebraic expression.

For an algebraic expression such as $2n - 3$, n could be any real number. That is, associated with any real number n, there is exactly one real number that is the value of $2n - 3$. However, if n and $2n - 3$ represent the number of cans of tuna that two customers buy, then n must be a whole number greater than or equal to 2 in order for both n and $2n - 3$ to be whole numbers.

For this situation, the domain or replacement set would be the set of whole numbers.

EXAMPLE 1

If n represents the number of points that Hradish scored in a basketball game and $2n - 3$ represents the number of points that his friend Brad scored, describe in words the number of points that Brad scored. What is a possible domain for the variable n?

Solution The number of points scored is always a whole number. In order that $2n - 3$ be a whole number, n must be at least 2.

Answer The number of points that Brad scored is 3 less than twice the number that Hradish scored. A possible domain for n is the set of whole numbers greater than or equal to 2.

EXAMPLE 2

Molly earned d dollars in July and $\frac{1}{2}d + 10$ dollars in August. Describe in words the number of dollars that Molly earned in August.

Answer In August, Molly earned 10 more than half the number of dollars that she earned in July.

EXAMPLE 3

Describe a situation in which x and $12 - x$ can be used to represent variable quantities. List the domain or replacement set for the answer.

Solution If x eggs are used from a full dozen of eggs, there will be $12 - x$ eggs left.

Answer The domain or replacement set is the set of whole numbers less than or equal to 12.

Another Solution The distance from my home to school is 12 miles. On my way to school, after I have traveled x miles, I have $12 - x$ miles left to travel.

Answer The domain or replacement set is the set of non-negative real numbers that are less than or equal to 12.

Many other answers are possible.

EXERCISES

Writing About Mathematics

1. a. If $4 + n$ represents the number of books Ken read in September and $4 - n$ represents the number of books he read in October, how many books did he read in these two months?

b. What is the domain of the variable n?

2. Pedro said that the replacement set for the amount that we pay for any item is the set of rational numbers of the form $0.01x$ where x is a whole number. Do you agree with Pedro? Explain why or why not.

Developing Skills

In 3–14: **a.** Write in words each of the given algebraic expressions. **b.** Describe a possible domain for each variable.

3. By one route, the distance that Ian walks to school is d miles. By a different route, the distance is $d - 0.2$ miles.

4. Juan pays n cents for a can of soda at the grocery store. When he buys soda from a machine, he pays $n + 15$ cents.

5. Yesterday Alexander spent a minutes on leisurely reading and $3a + 10$ minutes doing homework.

6. The width of a rectangle is w meters and the length is $2w + 8$ meters.

7. During a school day, Abby spends h hours in class, $\frac{h}{6}$ hours at lunch and $\frac{h}{3}$ hours on sports.

8. Jen spends d hours at work and $\frac{d}{12}$ hours driving to and from work.

9. Alicia's score for 18 holes of golf was g and her son's score was $10 + g$.

10. Tom paid d cents for a notebook and $5d + 30$ cents for a pen.

11. Seema's essay for English class had w words and Dominic's had $\frac{3}{4}w + 80$ words.

12. Virginia read r books last month and Anna read $3r - 5$ books.

13. Mario and Pete are playing a card game where it is possible to have a negative score. Pete's score is s and Mario's score is $s - 220$.

14. In the past month, Agatha has increased the time that she walks each day from m minutes to $3m - 10$ minutes.

3-5 EVALUATING ALGEBRAIC EXPRESSIONS

Benjamin has 1 more tape than 3 times the number of tapes that Julia has. If Julia has n tapes, then Benjamin has $3n + 1$ tapes.

The algebraic expression $3n + 1$ represents an unspecified number. Only when the variable n is replaced by a specific number does $3n + 1$ become a specific number. For example:

If $n = 10$, then $3n + 1 = 3(10) + 1 = 30 + 1 = 31$.
If $n = 15$, then $3n + 1 = 3(15) + 1 = 45 + 1 = 46$.

Since in this example, n represents the number of tapes that Julia has, only whole numbers are reasonable replacements for n. Therefore, the replacement set is the set of whole numbers or some subset of the set of whole numbers.

When we substitute specific values for the variables in an algebraic expression and then determine the value of the resulting expression, we are **evaluating the algebraic expression**.

When we determine the number that an algebraic expression represents for specific values of its variables, we are **evaluating the algebraic expression**.

Procedure

To evaluate an algebraic expression, replace the variables by the given values, and then follow the rules for the order of operations.

1. Replace the variables by the given values.
2. Evaluate the expression within the grouping symbols such as parentheses, always simplifying the expressions in the innermost groupings first.
3. Simplify all powers and roots.
4. Multiply and divide, from left to right.
5. Add and subtract, from left to right.

EXAMPLE 1

Evaluate $50 - 3x$ when $x = 7$.

Solution *How to Proceed*

(1) Write the expression: $50 - 3x$

(2) Replace the variable by its given value:	$50 - 3(7)$
(3) Multiply:	$50 - 21$
(4) Subtract:	29

Answer 29

EXAMPLE 2

Evaluate $2x^2 - 5x + 4$ when: **a.** $x = -7$ **b.** $x = 1.2$

Solution *How to Proceed*

a. (1) Write the expression:	$2x^2 - 5x + 4$
(2) Replace the variable by the value -7:	$2(-7)^2 - 5(-7) + 4$
(3) Evaluate the power:	$2(49) - 5(-7) + 4$
(4) Multiply:	$98 + 35 + 4$
(5) Add:	137
b. (1) Write the expression:	$2x^2 - 5x + 4$
(2) Replace the variable by the value 1.2:	$2(1.2)^2 - 5(1.2) + 4$
(3) Evaluate the power:	$2(1.44) - 5(1.2) + 4$
(4) Multiply:	$2.88 - 6 + 4$
(5) Add and subtract:	0.88

Answers **a.** 137 **b.** 0.88

EXAMPLE 3

Evaluate $\frac{2a}{5} + (n - 1)d$ when $a = -4$, $n = 10$, and $d = 3$.

Solution *How to Proceed*

(1) Write the expression:	$\frac{2a}{5} + (n - 1)d$
(2) Replace the variables with their given values:	$\frac{2(-4)}{5} + (10 - 1)(3)$
(3) Simplify the expressions grouped by parentheses or fraction bar:	$\frac{-8}{5} + (9)(3)$
(4) Multiply and divide:	$-1\frac{3}{5} + 27$
(5) Add:	$-1\frac{3}{5} + 26\frac{5}{5}$
	$25\frac{2}{5}$ *Answer*

Calculator Solution The values given for the variables can be stored in the calculator.

ENTER: (-) 4 STO▶ ALPHA A ENTER

10 STO▶ ALPHA N ENTER

3 STO▶ ALPHA D ENTER

DISPLAY:

```
-4→A
                 -4
10→N
                 10
3→D
                  3
```

Now enter the algebraic expression to be evaluated.

ENTER: 2 ALPHA A ÷ 5 + (ALPHA N − 1)

ALPHA D ENTER

DISPLAY:

```
2A/5+(N-1)D
               25.4
```

Answer $25\frac{2}{5} = 25.4$

EXAMPLE 4

Evaluate $(2x)^3 - 2x^3$ when $x = -0.40$.

Solution *How to Proceed*

(1) Write the expression:	$(2x)^3 - 2x^3$
(2) Replace the variable by its given value:	$[2(-0.40)]^3 - 2(-0.40)^3$
(3) Simplify the expression within brackets:	$[-0.80]^3 - 2(-0.40)^3$
(4) Evaluate the powers:	$-0.512 - 2(-0.064)$
(5) Multiply:	$-0.512 + 0.128$
(6) Subtract:	-0.384

Answer -0.384

EXERCISES

Writing About Mathematics

1. Explain why, in an algebraic expression such as $12ab$, 12 is called a constant and a and b are called variables?

2. Explain why, in step 2 of Example 1, parentheses were needed when x was replaced by its value.

Developing Skills

To understand this topic, you should first evaluate the expressions in Exercises 3 to 27 without a calculator. Then, store the values of the variables in the calculator and enter the given algebraic expressions to check your work.

In 3–27, find the numerical value of each expression. Use $a = 8$, $b = -6$, $d = 3$, $x = -4$, and $y = 0.5$.

3. $5a$ **4.** $\frac{1}{2}x$ **5.** $0.3y$ **6.** $a + 3$

7. $b - 2$ **8.** ax^2 **9.** $\frac{3bd}{9}$ **10.** $5x - 2y$

11. $7xy^3$ **12.** $ab - dx$ **13.** $\frac{2}{5}a + \frac{1}{5}b$ **14.** $0.2d + 0.3b$

15. $\frac{3}{4}x^3$ **16.** $(3y)^2$ **17.** $\frac{1}{4}x^2y$ **18.** $a^2 + 3d^2$

19. $(ay)^3$ **20.** $x(y - 2)$ **21.** $4(2x + 3y)$ **22.** $\frac{1}{2}x(y + 0.1)^2$

23. $3y - (x - d)$ **24.** $2(x + y) - 5$ **25.** $(x - d)^5$

26. $(2a - 5d)^2$ **27.** $(2a)^2 - (5d)^2$

Applying Skills

28. At one car rental agency, the cost of a car for one day can be determined by using the algebraic expression $\$32.00 + \$0.10m$ where m represents the number of miles driven. Determine the cost of rental for each of the following:

a. Mike Baier drove the car he rented for 35 miles.

b. Dana Morse drove the car he rented for 435 miles.

c. Jim Szalach drove the car he rented for 102 miles.

29. The local pottery co-op charges \$40.00 a year for membership and \$0.75 per pound for firing pottery pieces made by the members. The algebraic expression $40 + 0.75p$ represents the yearly cost to a member who brings p pounds of pottery to be fired. Determine the yearly cost for each of the following:

a. Tiffany is an amateur potter who fired 35 pounds of work this year.

b. Nia sells her pottery in a local craft shop and fired 485 pounds of work this year.

30. If a stone is thrown down into a deep gully with an initial velocity of 30 feet per second, the distance it has fallen, in feet, after t seconds can be found by using the algebraic expression $16t^2 + 30t$. Find the distance the stone has fallen:

a. after 1 second. **b.** after 2 seconds. **c.** after 3 seconds.

31. The Parkside Bread Company sells cookies and scones as well as bread. Bread (b) costs \$4.50 a loaf, cookies (c) cost \$1.10 each, and scones (s) cost \$1.50 each. The cost of a bakery order can be represented by $4.50b + 1.10c + 1.50s$. Determine the cost of each of the following orders:

a. six cookies and two scones

b. three loaves of bread and one cookie

c. one loaf of bread, a dozen cookies, and a half-dozen scones

32. A Green Thumb volunteer can plant shrubbery at a rate of 6 shrubs per hour and a Friendly Garden volunteer can plant shrubbery at a rate of 8 shrubs per hour. The total number of shrubs that g Green Thumb volunteers and f Friendly Garden volunteers can plant in h hours is given by the algebraic expression $6gh + 8fh$. Determine the number of shrubs planted:

a. in 3 hours by 2 Green Thumb and 1 Friendly Garden volunteers.

b. in 2 hours by 4 Green Thumb and 4 Friendly Garden volunteers.

3-6 OPEN SENTENCES AND SOLUTION SETS

In this chapter, you learned how to translate words into algebraic expressions. The value of an algebraic expression depends on the value of the variables. When the values of the variables change, the value of the algebraic expression changes. For example, $x + 6$ is an algebraic expression. The value of $x + 6$ depends on the value of x.

If one value is assigned to an algebraic expression, an algebraic sentence is formed. These sentences may be formulas, equations, or inequalities. For example, when the value 9 is assigned to the algebraic expression $x + 6$, we can write the sentence "Six more than x is 9." This sentence can be written in symbols as $x + 6 = 9$.

Every sentence that contains a variable is called an **open sentence**.

$x + 6 = 9 \qquad 3y = 12 \qquad 2n > 0 \qquad x + 5 \leq 8$

An open sentence is neither true nor false.

The sentence will be true or false only when the variables are replaced by numbers from a domain or a replacement set, such as {0, 1, 2, 3}.

The numbers from the domain that make the sentence *true* are the elements of the **solution set** of the open sentence. A solution set, as seen below, can contain one or more numbers or, at times, no numbers at all, from the replacement set.

EXAMPLE 1

Using the domain {0, 1, 2, 3}, find the solution set of each open sentence: **a.** $x + 6 = 9$ **b.** $2n > 0$

Solution **a.** *Procedure:* Replace x in the open sentence with numbers from the domain $\{0, 1, 2, 3\}$.

$x + 6 = 9$

Let $x = 0$.
Then $0 + 6 = 9$ is false.

Let $x = 1$.
Then $1 + 6 = 9$ is false.

Let $x = 2$.
Then $2 + 6 = 9$ is false.

Let $x = 3$.
Then $3 + 6 = 9$ is true.

Here, only when $x = 3$ does the open sentence become a true sentence.

Answer: **a.** Solution set = $\{3\}$.

b. *Procedure:* Replace n in the open sentence with numbers from the domain $\{0, 1, 2, 3\}$.

$2n > 0$

Let $n = 0$.
Then $2(0) > 0$ or $0 > 0$ is false.

Let $n = 1$.
Then $2(1) > 0$ or $2 > 0$ is true.

Let $n = 2$.
Then $2(2) > 0$ or $4 > 0$ is true.

Let $n = 3$.
Then $2(3) > 0$ or $6 > 0$ is true.

Here, three elements of the domain make the open sentence true.

Answer: **b.** Solution set = $\{1, 2, 3\}$

EXAMPLE 2

Find the solution set for the open sentence $3y = 12$ using:
a. the domain = $\{3, 5, 7\}$ **b.** the domain = {whole numbers}

Solution **a.** *Procedure:* Replace y with 3, 5, and 7.

If $y = 3$, then $3(3) = 12$ is false.

If $y = 5$, then $3(5) = 12$ is false.

If $y = 7$, then $3(7) = 12$ is false.

When y is replaced by each of the numbers from the domain, no true statement is found. The solution set *for this domain* is the empty set or the null set, written in symbols as { } or as $\varnothing$.

Answer: **a.** The solution set is { } or $\varnothing$.

b. *Procedure:* Of course, you cannot replace y with every whole number, but you can use multiplication facts learned previously.

You know that $3(4) = 12$. Let $y = 4$. Then $3(4) = 12$ is true. No other whole number would make the open sentence $3y = 12$ a true sentence.

Answer: **b.** The solution set is $\{4\}$.

EXERCISES

Writing About Mathematics

1. For the open sentence $x + 7 > 12$, write a domain for which the solution set is the empty set.

2. For the open sentence $x + 7 > 12$, write a domain for which the solution set has only one element.

3. For the open sentence $x + 7 > 12$, write a domain for which the solution set is an infinite set.

Developing Skills

In 4–11, tell whether each is an open sentence, a true sentence, or an algebraic expression.

4. $2 + 3 = 5 + 0$ **5.** $x + 10 = 14$ **6.** $y - 4$ **7.** $3 + 7 = 2(5)$

8. $n > 7$ **9.** $3 + 2 < 10$ **10.** $3r + 2$ **11.** $2x - 7 = 15$

In 12–15, name the variable in each open sentence.

12. $x + 5 = 9$ **13.** $4y = 20$ **14.** $r - 6 > 2$ **15.** $7 < 3 + a$

In 16–23, using the domain $\{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$, find the solution set for each open sentence.

16. $n + 3 = 7$ **17.** $x - x = 0$ **18.** $5 - n = 2$ **19.** $n + 3 > 9$

20. $2n + 1 < 8$ **21.** $\frac{2n+1}{3} = 4$ **22.** $\frac{3x}{2} < x$ **23.** $2x < -4$

Applying Skills

24. Pencils sell for \$0.19 each. Torry wants to buy at least one but not more than 10 pencils and has \$1.50 in his pocket.

a. Use the number of pencils that Torry wants to buy to write a domain for this problem.

b. The number of pencils that Torry might buy, x, can be found using the open sentence $0.19x \leq 1.50$. Find the solution set of this open sentence using the domain from part **a**.

c. How many pencils can Torry buy?

25. The local grocery store has frozen orange juice on sale for \$0.99 a can but limits the number of cans that a customer may buy at the sale price to no more than 5.

a. The domain for this problem is the number of cans of juice that a customer may buy at the sale price. Write the domain.

b. If Mrs. Dajhon does not want to spend more than \$10, the number of cans that she might buy at the sale price, y, is given by the equation $0.99y \leq 10$. Find the solution set of this equation using the domain from part **a**.

c. How many cans can Mrs. Dajhon buy if she does not want to spend more than \$10?

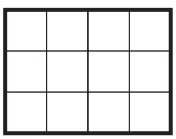
26. Admission to a recreation park is \$17.50. This includes all rides except for a ride called *The Bronco* that costs \$1.50 for each ride. Ian has \$25 to spend.

a. Find the domain for this problem, the number of times a person might ride *The Bronco*.

b. The number of times Ian might ride *The Bronco*, z, can be found using the open sentence $17.50 + 1.50z \leq 25$. Find the solution set of this open sentence using the domain from part **a**.

c. How many times can Ian ride *The Bronco*?

3-7 WRITING FORMULAS



A **formula** uses mathematical language to express the relationship between two or more variables. Some formulas are found by the strategy of looking for patterns. For example, how many square units are shown in the rectangle on the left? This rectangle, measuring 4 units in length and 3 units in width, contains a total of 12 square units of area.

Many such examples led to the conclusion that the area of a rectangle is equal to the product of its length and width. This relationship is expressed by the formula $A = lw$ where A, l, and w are variables that represent, respectively, the area, the length, and the width of a rectangle.

A formula is an open sentence that states that two algebraic expressions are equal. In formulas, the word *is* is translated into the symbol $=$.

EXAMPLE 1

Write a formula for each of the following relationships.

a. The perimeter P of a square is equal to 4 times the length of one side.

b. The total cost C of an article is equal to its price p plus an 8% tax on the price.

c. The sum S of the measures of the interior angles of an n-sided polygon is 180 times 2 less than the number of sides.

Solution **a.** Let s represent the length of each side of a square.

$$P = 4s$$ *Answer*

b. 8% (or 8 percent) means 8 hundredths, written as 0.08 or $\frac{8}{100}$.

$$C = p + 0.08p \text{ or } C = p + \frac{8}{100}p$$ *Answer*

c. "2 less than the number of sides" means $(n - 2)$.

$$S = 180(n - 2)$$ *Answer*

EXAMPLE 2

Write a formula that expresses the number of months m that are in y years.

Solution Look for a pattern.

In 1 year, there are 12 months.

In 2 years, there are 12(2) or 24 months.

In 3 years, there are 12(3) or 36 months.

In y years, there are $12(y)$ or $12y$ months. This equals m, the number of months.

Answer $m = 12y$

EXAMPLE 3

The Short Stop Diner pays employees \$6.00 an hour for working 40 hours a week or less. For working overtime, an employee is paid \$9.00 for each hour over 40 hours. Write a formula for the wages, W, of an employee who works h hours in a week.

Solution Two formulas are needed, one for $h \leq 40$ and the other for $h > 40$.

If $h \leq 40$, the wage is 6.00 times the number of hours, h. $W = 6.00h$

If $h > 40$, the employee has worked 40 hours at \$6.00 an hour and the remaining hours, $h - 40$, at \$9.00 an hour. $W = 6.00(40) + 9.00(h - 40)$.

Answer $W = 6h$ if $h \leq 40$ and $W = 6(40) + 9(h - 40)$ if $h > 40$.

(Note that the formula for $h > 40$ may also be given as $W = 240 + 9(h - 40)$.)

EXERCISES

Writing About Mathematics

1. Fran said that a recipe is a type of formula. Do you agree or disagree with Fran? Explain your answer.

2. a. Is an algebraic expression a formula? Explain why or why not.

b. Is a formula an open sentence? Explain why or why not.

Developing Skills

In 3–17, write a formula that expresses each relationship.

3. The total length l of 10 pieces of lumber, each m meters in length, is 10 times the length of each piece of lumber.

4. An article's selling price S equals its cost c plus the margin of profit m.

5. The perimeter P of a rectangle is equal to the sum of twice its length l and twice its width w.

6. The average m of three numbers, a, b, and c is their sum divided by 3.

7. The area A of a triangle is equal to one-half the length of the base b multiplied by the length of the altitude h.

8. The area A of a square is equal to the square of the length of a side s.

9. The volume V of a cube is equal to the cube of the length of an edge e.

10. The surface area S of a cube is equal to 6 times the square of the length of an edge e.

11. The surface area S of a sphere is equal to the product of 4π and the square of the radius r.

12. The average rate of speed r is equal to the distance that is traveled d divided by the time spent on the trip t.

13. The Fahrenheit temperature F is 32° more than nine-fifths of the Celsius temperature C.

14. The Celsius temperature C is equal to five-ninths of the difference between the Fahrenheit temperature F and 32°.

15. The dividend D equals the product of the divisor d and the quotient q plus the remainder r.

16. A sales tax T that must be paid when an article is purchased is equal to 8% of the price of the article v.

17. A salesman's weekly earnings F is equal to his weekly salary s increased by 2% of his total volume of sales v.

Applying Skills

18. A ferry takes cars, drivers, and passengers across a body of water. The total ferry charge C in dollars is \$20.00 for the car and driver, plus d dollars for each passenger.

a. Write a formula for C in terms of d and the number of passengers, n.

b. Find the cost of the ferry for a car if $d =$ \$15 and there are 5 persons in the car.

c. Find the cost of the ferry for a car with only the driver.

19. The cost C in cents of an internet telephone call lasting m minutes is x cents for the first 3 minutes and y cents for each additional minute.

a. Write two formulas for C, one for the cost of calls lasting 3 minutes or less ($m \leq 3$), and another for the cost of calls lasting more than 3 minutes ($m > 3$).

b. Find the cost of a 2.5 minute telephone call if $x = \$0.25$ and $y = \$0.05$.

c. Find the cost of a 10 minute telephone call if $x = \$0.25$ and $y = \$0.05$.

20. The cost D in dollars of sending a fax of p pages is a dollars for sending the first page and b dollars for each additional page.

a. Write two formulas for D, one for the cost of faxing 1 page ($p = 1$), and another for the cost of faxing more than 1 page ($p > 1$).

b. Find the cost of faxing 1 page if $a = \$1.00$ and $b = \$0.60$.

c. Find the cost of faxing 5 pages if $a = \$1.00$ and $b = \$0.60$.

21. A gasoline dealer is allowed a profit of 12 cents a gallon for each gallon sold. If more than 25,000 gallons are sold in a month, an additional profit of 3 cents for every gallon over that number is given.

a. Write two formulas for the gasoline dealer's profit, P, one for when the number of gallons sold, n, is not more than 25,000 ($n \leq 25{,}000$), and another for when more than 25,000 gallons are sold ($n > 25{,}000$).

b. Find P when 21,000 gallons of gasoline are sold in one month.

c. Find P when 30,000 gallons of gasoline are sold in one month.

22. Gabriel earns a bonus of \$25 for each sale that he makes if the number of sales, s, in a month is 20 or less. He earns an extra \$40 for each additional sale if he makes more than 20 sales in a month.

a. Write a formula for Gabriel's bonus, B, when $s \leq 20$.

b. Write a formula for B when $s > 20$.

c. In August, Gabriel made 18 sales. Find his bonus for August.

d. In September, Gabriel made 25 sales. Find his bonus for September.

23. Mrs. Lucy is selling cookies at a local bake sale. If she sells exactly 3 dozen cookies, the cost of ingredients will equal her earnings. If she sells more than 3 dozen cookies, Mrs. Lucy will make a profit of 25 cents for each cookie sold.

a. Write a formula for Mrs. Lucy's earnings, E, when the number of cookies sold, c, is equal to 36.

b. Write a formula for E when $c > 36$.

c. Find the number of cookies Mrs. Lucy sold if she makes a profit of \$2.00.

CHAPTER SUMMARY

An **algebraic expression**, such as $x + 6$, is an expression or a phrase that contains one or more variables, such as x. The **variable** is a placeholder for numbers. To evaluate an expression, replace each variable with a number and follow the order of operations.

A **term** is a number, a variable, or any product or quotient of numbers and variables. In the term $6by$, 6 is the **numerical coefficient**. In the term n^3, the **base** is n, the **exponent** is 3, and the **power** is n^3. The power n^3 means that base n is used as a factor 3 times.

An **open sentence**, which can be an equation or an inequality, contains a variable. When the variable is replaced by numbers from a **domain**, the numbers that make the open sentence true are the elements of the **solution set** of the sentence.

A **formula** is a sentence that shows the relationship between two or more variables.

VOCABULARY

3-1 Variable • Placeholder • Domain • Replacement set • Algebraic expression

3-3 Term • Factor • Coefficient • Numerical coefficient

3-5 Evaluating an algebraic expression

3-6 Open sentence • Solution set

3-7 Formula

REVIEW EXERCISES

1. Explain the difference between an algebraic expression and an open sentence.

2. Explain the difference between $2a^2$ and $(2a)^2$.

In 3–6, use mathematical symbols to translate the verbal phrases into algebraic language.

3. x divided by b

4. 4 less than r

5. q decreased by d

6. 3 more than twice g

In 7–14, find the value of each expression.

7. $6ac - d$ when $a = 10$, $c = 8$, and $d = 5$

8. $4b^2$ when $b = 2.5$

9. $3b + c$ when $b = 7$ and $c = 14$

10. $km - 9$ when $k = 15$ and $m = 0.6$

11. $\frac{bc}{a}$ when $a = 5, b = 3$, and $c = 12$

12. $2a^2 - 2a$ when $a = \frac{1}{4}$

13. $(2a)^2 - 2a$ when $a = \frac{1}{4}$

14. $a(b + c)$ when $a = 2.5, b = 1.1$, and $c = 8.9$

15. Write an algebraic expression for the total number of cents in n nickels and q quarters.

16. In the term $2xy^3$ what is the coefficient?

17. In the term $2xy^3$ what is the exponent of y?

18. In the term $2xy^3$, what is the base that is used 3 times as a factor?

19. If distance is the product of rate and time, write a formula for distance, d, in terms of rate, r, and time, t.

20. What is the smallest member of the solution set of $19.4 \leq y - 29$ if the domain is $\{46.25, 47.9, 48, 48.5, 49, 50, 51.\overline{3}\}$?

21. What is the smallest member of the solution set of $19.4 \leq y - 29$ if the domain is the set of whole numbers?

22. What is the smallest member of the solution set of $19.4 \leq y - 29$ if the domain is the set of real numbers?

23. In a baseball game, the winning team scored n runs and the losing team scored $2n - 5$ runs.

a. Describe in words the number of runs that the losing team scored.

b. What could have been the score of the game? Is there more than one answer?

c. What are the possible values for n?

24. A mail order book club offers books for \$8.98 each plus \$3.50 for shipping and handling on each order. The cost of Bethany's order, which totaled less than \$20, can be expressed as $8.98b + 3.50 < 20$ where b represents the number of books Bethany ordered.

a. What could be the domain for this problem?

b. What is the solution set for this open sentence?

25. Write two algebraic expressions to represent the cost of sending an express delivery based on the rates given in the chapter opener on page 88, the first if the delivery distance is 10 miles or less, and the second if the delivery distance is more than 10 miles.

26. A list of numbers that follows a pattern begins with the numbers 2, 5, 8, 11, ...

a. Find the next number in the list.

b. Write a rule or explain how the next number is determined.

c. What is the 25th number in the list?

27. Each of the numbers given below is different from the others, that is, it belongs to a set of numbers to which the others do not belong. Explain why each is different.

3 6 9 35

28. Two oranges cost as much as five bananas. One orange costs the same as a banana and an apple. How many apples cost the same as three bananas?

Exploration

STEP 1. Write a three-digit multiple of 11 by multiplying any whole number from 10 to 90 by 11. Add the digits in the hundreds and the ones places. If the sum is greater than or equal to 11, subtract 11. Compare this result to the digit in the tens place. Repeat the procedure for other three-digit multiples of 11.

STEP 2. Write a three-digit number that is not a multiple of 11 by adding any counting number less than 11 to a multiple of 11 used in step 1. Add the digits in the hundreds and the ones places. If the sum is greater than or equal to 11, subtract 11. Compare this result to the digit in the tens place. Repeat the procedure with another number.

STEP 3. Based on steps 1 and 2, can you suggest a way of determining whether or not a three-digit number is divisible by 11?

STEP 4. Write a four-digit multiple of 11 by multiplying any whole number from 91 to 909 by 11. Add the digits in the hundreds and ones places. Add the digits in the thousands and tens places. If one sum is greater than or equal to 11, subtract 11. Compare these results. Repeat the procedure for another four-digit multiple of 11.

STEP 5. Write a four-digit number that is not a multiple of 11 by adding any counting number less than 11 to a multiple of 11 used in step 4. Add the digits in the hundreds place and ones place. Add the digits in the thousands place and tens place. If one sum is greater than or equal to 11, subtract 11. Compare these results. Repeat the procedure starting with another number.

STEP 6. Based on steps 4 and 5, can you suggest a way of determining whether or not a four-digit number is divisible by 11?

STEP 7. Write a rule for determining whether or not any whole number is divisible by 11.

CUMULATIVE REVIEW — CHAPTERS 1–3

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

1. Which of the following is the set of negative integers greater than –3?
(1) $\{-4, -5, -6, -7, \ldots\}$ (3) $\{-3, -2, -1\}$
(2) $\{-3, -4, -5, -6, \ldots\}$ (4) $\{-2, -1\}$

2. The exact value of the rational number $\frac{5}{3}$ can be written as
(1) 1.6 (2) $1.\overline{6}$ (3) 1.666666667 (4) 1.666666666

3. Rounded to the nearest hundredth, $\sqrt{5}$ is approximately equal to
(1) 2.23 (2) 2.236 (3) 2.24 (4) 2.240

4. Which of the following numbers is rational?
(1) π (2) $\sqrt{2}$ (3) $1.4\overline{2}$ (4) $\sqrt{0.4}$

5. Which of the following inequalities is a true statement?
(1) $0.026 < 0.25 < 0.2$ (2) $0.2 < 0.026 < 0.25$
(3) $0.2 < 0.25 < 0.026$ (4) $0.026 < 0.2 < 0.25$

6. The length of a rectangle is given as 30.02 yards. This measure has how many significant digits?
(1) 1 (2) 2 (3) 3 (4) 4

7. Which of the following is not a prime?
(1) 7 (2) 23 (3) 37 (4) 51

8. The arithmetic expression $8 - 5(-0.2)^2 \div 10$ is equal to
(1) 0.9 (2) 0.78 (3) 7.98 (4) 8.02

9. Which of the following is a correct application of the distributive property?
(1) $4(8 + 0.2) = 4(8) + 4(0.2)$ (3) $8(5 + 4) = 8(5) + 4$
(2) $6(3 - 1) = 6(-1 + 3)$ (4) $8(5)(4) = 8(5) + 8(4)$

10. The additive inverse of 7 is
(1) –7 (2) 0 (3) $\frac{1}{7}$ (4) $|7|$

Part II

Answer all questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit.

11. Of the 80 students questioned about what they had read in the past month, 35 had read nonfiction, 55 had read fiction, and 22 had read neither fiction nor nonfiction. How many students had read both fiction and nonfiction?

12. What is the largest number that is the product of three different two-digit primes?

Part III

Answer all questions in this part. Each correct answer will receive 3 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit.

13. The formula for the area of an equilateral triangle (a triangle with three sides of equal measure) is $A = \frac{\sqrt{3}}{4}s^2$. Find the area of an equilateral triangle if the measure of one side is 12.6 centimeters. Express your answer to the number of significant digits determined by the given data.

14. A teacher wrote the sequence 1, 2, 4, . . . and asked what the next number could be. Three students each gave a different answer and the teacher said that all three answers were correct.

a. Adam said 7. Explain what rule Adam used.

b. Bette said 8. Explain what rule Bette used.

c. Carlos said 5. Explain what rule Carlos used.

Part IV

Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit.

15. If $a = 7$, $b = -5$, and $c = \frac{1}{3}$, evaluate the expression $\frac{a-b}{c} + 3(b - 2)$. Do not use a calculator. Show each step in your calculation.

16. Michelle bought material to make a vest and skirt. She used half of the material to make the skirt and two-thirds of what remained to make the vest. She had $1\frac{1}{4}$ yards of material left.

a. How many yards of material did she buy?

b. How many yards of material did she use for the vest?

c. How many yards of material did she use for the skirt?